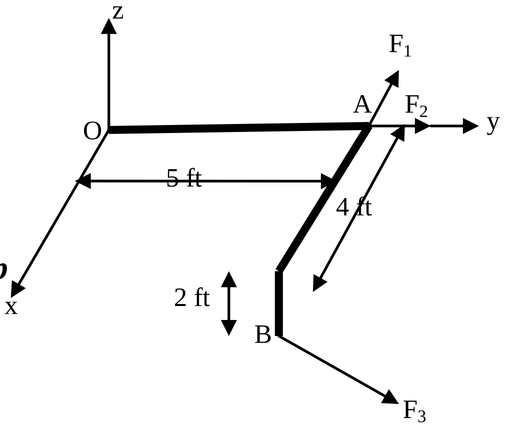
عي

Benha University
Faculty of Engineering- Shoubra
Mathematics and Engineering
Physics Department
Year: Preparatory

Final 2nd Term Exam
Date: 31 / 5 / 2014
Titel:Mechanics
Code: ENP 012
Duration : 4 hours

- Answer all the following questions
- Illustrate your answers with sketches when necessary.
- The exam. consists of two pages
- No. of Questions:…6..
- Total Mark: 120…. Marks

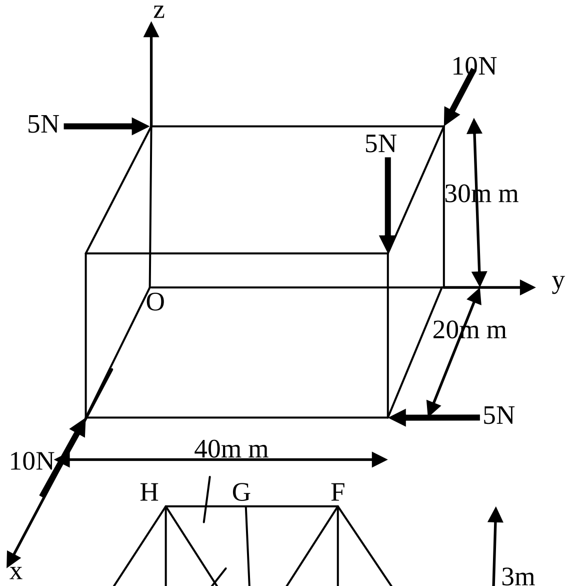
Question No. 1 (20 Marks) [a_1-a_2-b_2]



(a) Three forces act on the shown rod . Determine the resultant moment at the point (o) , and determine the direction of this moment axis .

$\overrightarrow{F_1} = (-60, 40, 20)lb$, $\overrightarrow{F_2} = (0, 50, 0)lb$, $\overrightarrow{F_3} = (80, 40, -30)lb$

(10 marks)

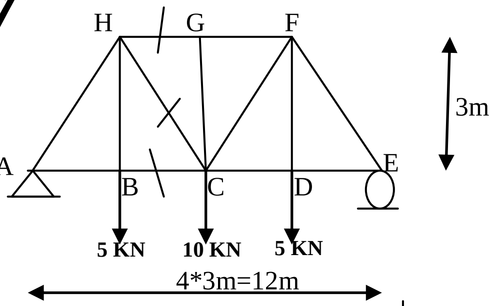


(b) A rectangular block is acted upon by the five forces shown , which are directed along the edges. Reduce the system of forces to :

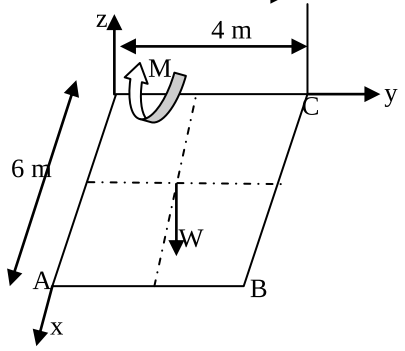
1. a force and a couple at (o)

2. a wrench (specify its pitch and axis) (10 marks)

Question No. 2 (20 Marks)[a_1-b_1]



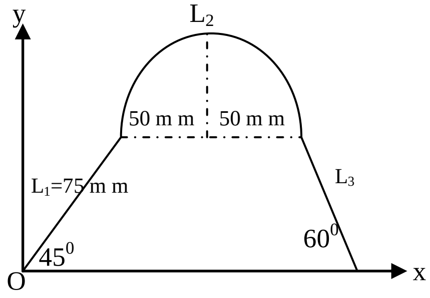
.(a) Using the method of sections , determine the force in indicated menbers for the given truss . (10 marks)



(b) The homogeneous plate shown in fig. has a weight W = 981 N , and is subjected to a force and a couple moment M = 200 N.m along its edge . It is supprted in a horizontal plane by means of a roller at A , a ball and socket at B , and a cord at C . Determine the components of reactions at supports (10 marks)

Question No. 3 (20 Marks)[a_1-a_2-b_3]

Locate the centroid of the bent wire shown in fig.



Question No. 4 (20 Marks)[a_1-a_2-c_1]

(a) A point describes a circle of equation : r = 2 a cos θ , and of radius (a) with uniform speed v . Show that the radial and transverse accelerations are $-\frac{v^2}{a}\cos\theta , -\frac{v^2}{a}\sin\theta$, a diameter is taken as initial line and one end of the diameter as the pole . (10 marks)

(b) A particle is projected with velocity (u) , so that the range on the horizontal plane is twice the gratest height . Prove that the range is $\frac{4\,u^2}{5\,g}$ (10 marks)

Question No. 5 (20 Marks)[a_1-b_2-c_1-d_1]

A particle of mass (m) is suspended from a fixed point (A) by an elastic string of natural length (L) , and modulus of elasticity (4 m g) , prove that the maximum depth is (2 L) downward the point (A) , Also prove that the time needed to cover this distance is :

$$\sqrt{\frac{L}{g}}[\sqrt{2}+\frac{\pi}{4}+\frac{1}{2}\sin^{-1}\left(\frac{1}{3}\right)]$$

Question No. 6 (20 Marks)[a_1-a_2-b_3-c_1-d_1]

(a) If the force $\vec{F} = (y^2+2xy)\hat{\imath} + (2xy+x^2)\hat{\jmath}$, find :
1. the work done by this force along the broken line (0,0) to (2,0) and from (2,0) to (2,4)
2. verify that the potential function of this force is : U (x,y) = - (xy^2 + x^2y) + c (10 marks)

(b) A particle is attached to the end of a string of length (a) is projected horizontally with velocity $\sqrt{nga}$. Prove that it rises to a height $\frac{n+1}{3}\,a$ before the string become slack .

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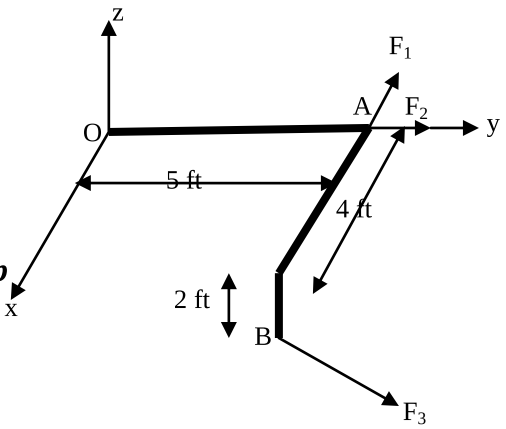
Question No. 1 (20 Marks) [a_1-a_2-b_2]



(a) Three forces act on the shown rod . Determine the resultant moment at the point (o) , and determine the direction of this moment axis .

$\overrightarrow{F_1} = (-60, 40, 20)lb$, $\overrightarrow{F_2} = (0, 50, 0)lb$, $\overrightarrow{F_3} = (80, 40, -30)lb$

(10 marks)

$$\overrightarrow{F_1} = (-60, 40, 20)lb$$
$$\overrightarrow{F_2} = (0, 50, 0)lb$$
$$\overrightarrow{F_3} = (80, 40, -30)lb$$
$$\overrightarrow{M_O} = \overrightarrow{OA} \times \overrightarrow{F_1} + \overrightarrow{OA} \times \overrightarrow{F_2} + \overrightarrow{OB} \times \overrightarrow{F_3}$$
$$\overrightarrow{M_O} = \overrightarrow{OA} \times (\overrightarrow{F_1} + \overrightarrow{F_2}) + \overrightarrow{OB} \times \overrightarrow{F_3}$$
$$\overrightarrow{M_O} = \begin{vmatrix} \hat{i} & j & k \\ 0 & 5 & 0 \\ -60 & 90 & 20 \end{vmatrix} + \begin{vmatrix} \hat{i} & j & k \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$
$$\overrightarrow{M_O} = (100, 0, 300) + (-70, -40, -240)$$
$$\overrightarrow{M_O} = (30, -40, 60) lb.ft$$
$$M_O = \sqrt{(30)^2 + (-40)^2 + (60)^2} = 10\sqrt{61} lb.ft$$
$$M_O = \frac{\overrightarrow{M_O}}{M_O} = \frac{(30, -40, 60)}{10\sqrt{61}} = \frac{(3, -4, 6)}{\sqrt{61}}$$

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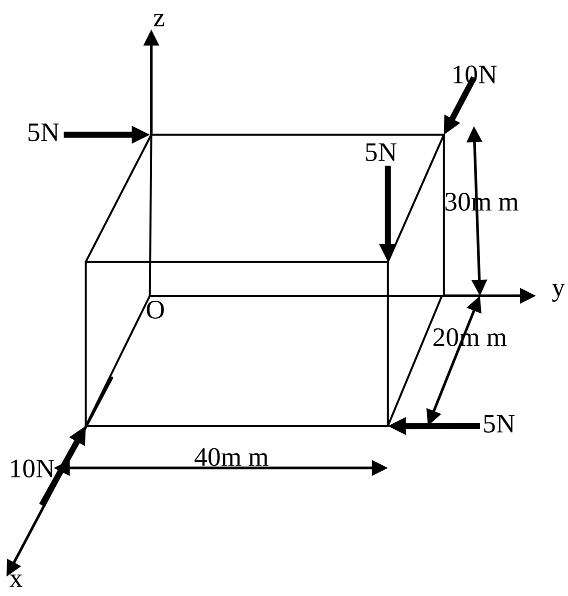
(b) A rectangular block is acted upon by the five forces shown , which are directed along the edges. Reduce the system of forces to :
1. a force and a couple at (o)
2. a wrench (specify its pitch and axis) (10 marks)



$$\overrightarrow{F_1} = (-10, 0, 0)N$$
$$\overrightarrow{F_2} = (0, -5, 0)N$$
$$\overrightarrow{F_3} = (0, 0, -5)N$$
$$\overrightarrow{F_4} = (0, 5, 0)N$$
$$\overrightarrow{F_5} = (10, 0, 0)N$$
$$\overrightarrow{R} = (-10, 0, -5)N$$

$$\overrightarrow{M_O} = \vec{0} + \begin{vmatrix} \hat{i} & j & k \\ 20 & 40 & 0 \\ 0 & -5 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & j & k \\ 20 & 40 & 30 \\ 0 & 0 & -5 \end{vmatrix}$$

$$+ \begin{vmatrix} \hat{i} & j & k \\ 0 & 0 & 30 \\ 0 & 5 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & j & k \\ 0 & 40 & 30 \\ 10 & 0 & 0 \end{vmatrix}$$

$$\overrightarrow{M_O} = (0, 0, -100) + (-200, -100, 0)$$
$$+(-150, 0, 0) + (0, 300, -400)$$
$$\overrightarrow{M_O} = (-350, 200, -500)N.m$$

$$R = \sqrt{(-10)^2 + (0)^2 + (-5)^2} = 5\sqrt{2}N$$

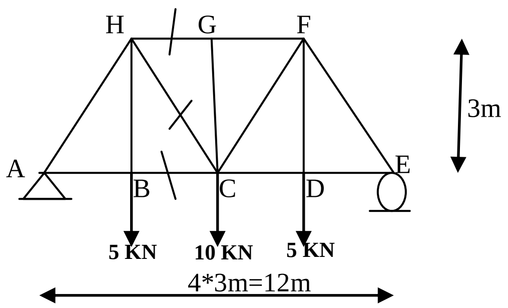
$$S = \frac{\overrightarrow{R} \bullet \overrightarrow{M_O}}{R^2} = \frac{(-10, 0, -5) \bullet (-350, 200, -500)}{125} = 48m$$

$$\vec{r} = \frac{\overrightarrow{R} \times \overrightarrow{M_O}}{R^2} + \lambda\overrightarrow{R} = \frac{1}{125} \begin{vmatrix} \hat{i} & j & k \\ -10 & 0 & -5 \\ -350 & 200 & 500 \end{vmatrix} + \lambda\overrightarrow{R}$$

$$= \frac{1}{125}(1000, 6750, -2000) + \lambda\overrightarrow{R}$$
$$= (8, 54, -16) + \lambda(-10, 0, -5)$$

Question No. 2 (20 Marks)[a_1-b_1]

.(a) Using the method of sections , determine the force in indicated menbers for the given truss . (10 marks)



$$X_A = 0, Y_A = Y_B = 10KN$$

$$\text{sec.}: S - S, left$$

$$\sum Y = 0$$

$$10 + 5 - \frac{F_{HC}}{\sqrt{2}} = 0$$

$$F_{HC} = 15\sqrt{2}KN(T)$$

$$\sum M_H = 0$$

$$-(10)(3) - F_{BC}(3) = 0$$

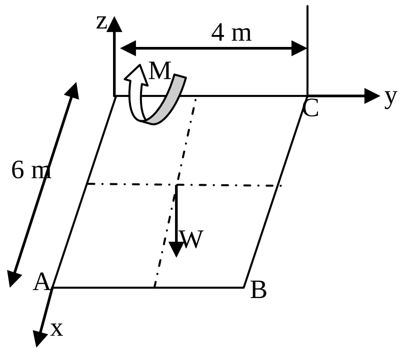
$$F_{BC} = -10KN(C)$$

$$\sum X = 0$$

$$-10 + 15\sqrt{2} \times \frac{1}{\sqrt{2}} + F_{HG} = 0$$

$$F_{HG} = -5KN(C)$$

(b) The homogeneous plate shown in fig. has a weight W = 981 N , and is subjected to a force and a couple moment M = 200 N.m along its edge . It is supprted in a horizontal plane by means of a roller at A , a ball and socket at B , and a cord at C . Determine the components of reactions at supports (10 marks)



$$X = 0 \rightarrow B_x = 0$$

$$Y = 0 \rightarrow B_y = 0$$

$$Z = 0 \rightarrow A_z + B_z + T = 981N$$

$$M_{CB} = 0 \rightarrow -A_z(4) + 981(2) = 0$$

$$A_z = 490.5N$$

$$M_y = 0$$

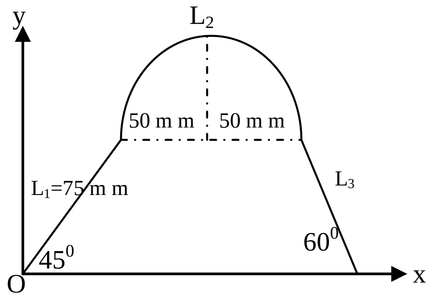
$$-200 - A_z(6) - B_z(6) + 981(3) = 0$$

$$B_z = 0$$

$$T = 490.5N$$

Question No. 3 (20 Marks)[a_1-a_2-b_3]

Locate the centroid of the bent wire shown in fig.



$$L_1 = 75mm, G_1 = (53, 53)mm$$
$$L_2 = 157mm, G_1 = (156, 138)mm$$
$$L_1 = 122mm, G_1 = (236.5, 70.4)mm$$
$$\overline{X} = \frac{1}{354}(75\times53 + 157\times156 + 122\times236.5) = 161.9mm$$
$$\overline{Y} = \frac{1}{354}(75\times53 + 157\times138 + 122\times70.4) = 96.7mm$$

Question No. 4 (20 Marks)[a_1-a_2-c_1]

(a) A point describes a circle of equation : r = 2 a cos θ , and of radius (a) with uniform speed v . Show that the radial and transverse accelerations are $-\frac{v^2}{a}\cos\theta, -\frac{v^2}{a}\sin\theta$, a diameter is taken as initial line and one end of the diameter as the pole . (10 marks)

$$V_r = -v\sin\theta$$
$$V_\theta = v\cos\theta$$
$$\dot{r} = -v\sin\theta$$
$$r\dot{\theta} = v\cos\theta$$
$$\dot{\theta} = \frac{v\cos\theta}{2a\cos\theta} = \frac{v}{2a} = cons\tan t$$
$$\ddot{r} = -v\cos\theta \times \dot{\theta} = -v\cos\theta \times (\frac{v}{2a}) = -\frac{v^2}{2a}\cos\theta$$
$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{v^2}{2a}\cos\theta - 2a\cos\theta(\frac{v^2}{4a^2}) = -\frac{v^2}{a}\cos\theta$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-v\sin\theta)(\frac{v}{2a}) = -\frac{v^2}{a}\sin\theta$$

(b) A particle is projected with velocity (u) , so that the range on the horizontal plane is twice the gratest height . Prove that the range is $\frac{4\,u^2}{5\,g}$ (10 marks)

$R = 2h$

$\frac{2u^2 \sin\alpha \cos\alpha}{g} = 2\frac{u^2 \sin^2\alpha}{2g}$

$2\cos\alpha = \sin\alpha$

$\tan\alpha = \frac{1}{2}$

$R = \frac{2u^2 \sin\alpha \cos\alpha}{g}$

$R = \frac{2u^2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}}{g} = \frac{4u^2}{5g}$

Question No. 5 (20 Marks)[a_1-b_2-c_1-d_1]

A particle of mass (m) is suspended from a fixed point (A) by an elastic string of natural length (L) , and modulus of elasticity (4 m g) , prove that the maximum depth is (2 L) downward the point (A) , Also prove that the time needed to cover this distance is :
$\sqrt{\frac{L}{g}}[\sqrt{2}+\frac{\pi}{4}+\frac{1}{2}\sin^{-1}\left(\frac{1}{3}\right)]$

In the static position:

$T_0=\frac{\lambda}{L}(L_1-L)=mg \rightarrow mg=\frac{4\,mg}{L}\Delta L \rightarrow \Delta L=\frac{L}{4}$

Motion from a→b

$v_b^2=v_A^2+2gh=2gL \rightarrow v_b=\sqrt{2gL}$

$v_b=v_A+gt \rightarrow \sqrt{2gL}=gt \rightarrow t_{AB}=\sqrt{\frac{2L}{g}}$

Motion from b→c

Mg-T=m$\ddot{X}$

$-\frac{\lambda}{L}[X+\Delta L]+mg=\mathrm{m}\ddot{X}$

$-\frac{\lambda}{L}X-mg+mg=\mathrm{m}\ddot{X}$

$-\frac{\lambda}{L}X=\mathrm{m}\ddot{X}$

$-\frac{4\,mg}{L}X=\mathrm{m}\ddot{X}$

$\ddot{X}=-\frac{4\,g}{L}X=-\omega^2\mathrm{X}$

$\omega=\sqrt{\frac{4\,g}{L}}$

At b: $v^2=\omega^2[A^2-X^2]$

2gL=$\frac{4\,g}{L}[A^2-(\Delta L)^2]$

$\frac{L^2}{2}=A^2-\frac{L^2}{16}$

$A = \frac{3}{4}L$

ad= L + (L\4) +(3L\4) = 2 L

$t_{ab} = \sqrt{2}\sqrt{\frac{L}{g}}$

$\Theta = \omega t$

$\frac{\pi}{2} + sin^{-1}\frac{1}{3} = 2\sqrt{\frac{g}{L}}\ t_{bd}$

$t_{bd} = \frac{1}{2}\sqrt{\frac{L}{g}}(\frac{\pi}{2} + sin^{-1}\frac{1}{3})$

$T = \sqrt{\frac{L}{g}}[\sqrt{2} + \frac{\pi}{4} + \frac{1}{2}sin^{-1}\left(\frac{1}{3}\right)]$

Question No. 6 (20 Marks)[a_1-a_2-b_3-c_1-d_1]

(a) If the force $\vec{F} = (y^2 + 2xy)\hat{i} + (2xy + x^2)\hat{j}$, find :
1. the work done by this force along the broken line (0,0) to (2,0) and from (2,0) to (2,4)
2. verify that the potential function of this force is : U (x,y) = - ($xy^2 + x^2y$) + c (10 marks)

$$W_1 = \int_{(0,0)}^{(2,0)} [(y^2 + 2xy)\hat{i} + (2xy + x^2)\, j] \bullet \overrightarrow{dr}$$

$$= \int_{(0,0)}^{(2,0)} [(y^2 + 2xy)\hat{i} + (2xy + x^2)\, j](dx\hat{i} + dy\, j)$$

$$= \int_{(0,0)}^{(2,0)} [(x^2)\, j](dx\hat{i}) = 0$$

$$W_2 = \int_{(2,0)}^{(2,4)} [(y^2 + 2xy)\hat{i} + (2xy + x^2)\, j] \bullet \overrightarrow{dr}$$

$$= \int_{(2,0)}^{(2,4)} [(y^2 + 4y)\hat{i} + (4y + 4)\, j] \bullet (dy\, j)$$

$$= (2y^2 + 4y)_0^4 = 32 + 16 = 48$$

$$\frac{\partial F_x}{\partial y} = 2y + 2x, \frac{\partial F_y}{\partial x} = 2y + 2x$$

$$\therefore \frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \rightarrow \cos ervative$$

$$F_x = -\frac{\partial U}{\partial x} \rightarrow y^2 + 2xy = -\frac{\partial U}{\partial x}$$

$$U = -\int (y^2 + 2xy)\partial x = -(y^2x + x^2y) + f(y)$$

$$\frac{\partial U}{\partial y} = -F_y$$

$$-(2xy + x^2) + f' = -(2xy + x^2) \rightarrow f' = 0$$

$$f(y) = C$$

$$U = -(y^2x + x^2y) + C$$

(b) A particle is attached to the end of a string of length (a) is projected horizontally with velocity $\sqrt{nga}$. Prove that it rises to a height $\frac{n+1}{3}$ a before the string become slack .

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = -mgh$$

$$v_B^2 - nga = -2gh$$

$$v_B^2 = nga - 2g(a + a\cos\theta)$$

$$at, B:$$

$$mg\cos\theta = \frac{mv_B^2}{a}$$

$$v_B^2 = ga\cos\theta$$

$$ga\cos\theta = nga - 2g(a + a\cos\theta)$$

$$3\cos\theta = n - 2$$

$$\cos\theta = \frac{n-2}{3}$$

$$h = a + a\frac{n-2}{3} = \frac{3a + na - 2a}{3} = \frac{a + na}{3} = \frac{1+n}{3}a$$

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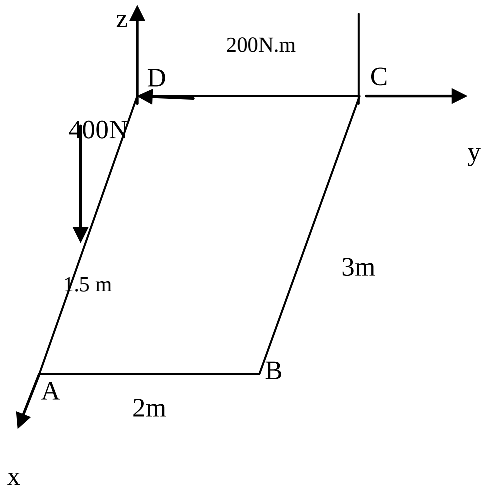
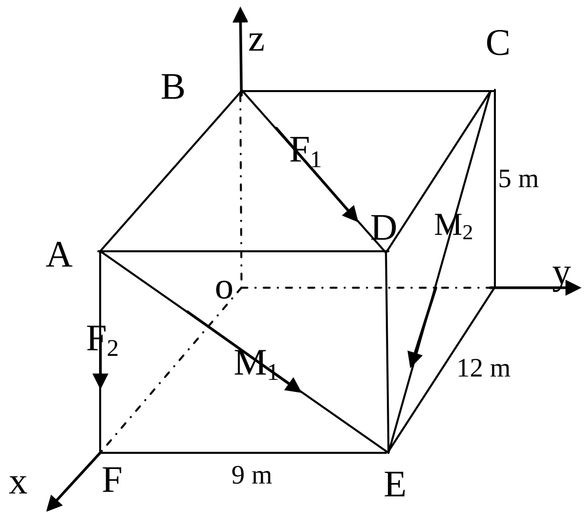


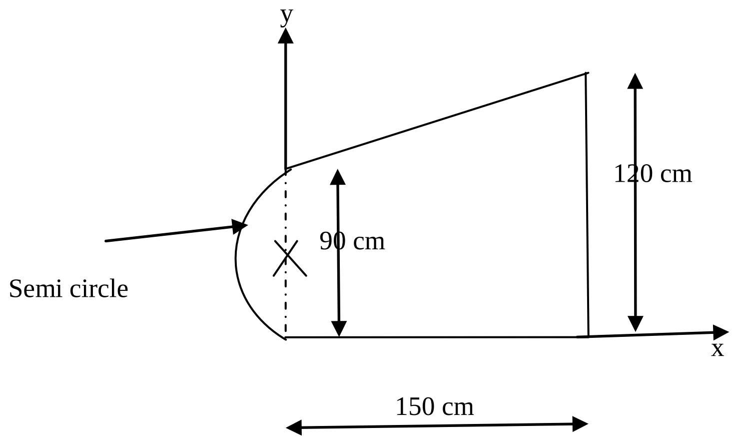
Question No. 2 (20 Marks)[a_1-b_1]

.

(a) Replace the set of forces and couples shown by a wrench at (o) .
F_1 = 30 N
F_2 = 20 N
$M_1 = 20\sqrt{106}$ N.m
M_2 = 156 N.m **(10 marks)**



(b) Determine the centroid for the shown area . **(10 marks)**



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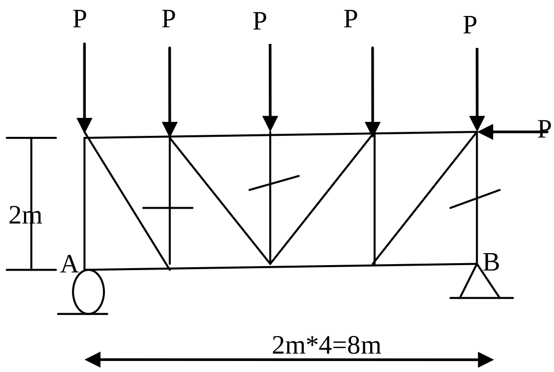
Question No. 3 (20 Marks)[a_1-a_2-b_3]

Determine forces in marked members for the shown truss.
P=100 N

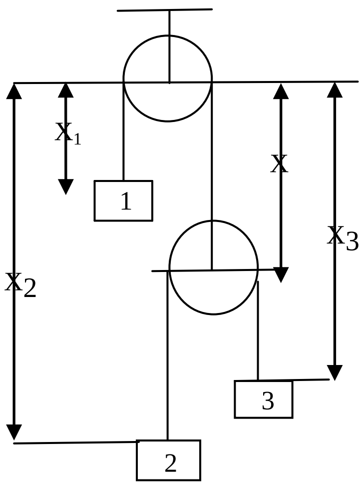


Question No. 4 (20 Marks)[a_1-a_2-c_1]

In the system shown , determine the velocity and acceleration of block (2) at the instant. Knowing that:

$\dot{X}_1 = 6\ m\backslash sec.\uparrow$, $\ddot{X}_1 = 2\ m\backslash sec.^2 \downarrow$

$\dot{X}_3 = 3\ m\backslash sec.\uparrow$, $\ddot{X}_3 = 4\ m\backslash sec.^2 \downarrow$



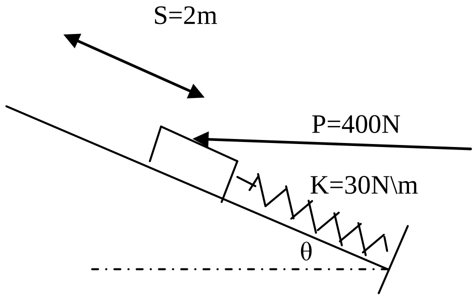
Question No. 5 (20 Marks)[a_1-b_2-c_1-d_1]

A light elastic string of natural length (L) has one extremity fixed at point (A) and the other end is attached to a stone , the weight of which in equilibrium position would extend the string to a length (L_1). Shoe that if the stone be dropped from rest at A , it will come to instantaneous rest at a depth $\sqrt{{L_1}^2 - L^2}$ below the equilibrium position and this depth attained in time

$$\sqrt{\frac{2L}{g}} + \sqrt{\frac{L_1 - L}{g}}\left[\pi - \cos^{-1}\left(\frac{L_1 - L}{L_1 + L}\right)\right]$$

Question No. 6 (20 Marks)[a_1-a_2-b_3-c_1-d_1]

(a) The 10 kg block in the figure rests on the smooth inclined plane . If the spring is originally stretched 0.5 m . Determine the total work done by all the forces acting on the block When a horizontal force P=400 N pushes the block up the plane S=2m , (K=30N\m) , θ=30^0



(b) A square table ABCD whose side is (a) , has raised edges. A particle of elasticity (e) is projected from a point (p) on A\B and hits the sides AB, BC, CD and DA at Q, R and S . Prove that PQ and RS are parallel. If (α) be the angle QPB and BP=x , prve that if the particle returns to P, then:
X(1-e)=α(1-e cot α) .

مع التمنيات بالنجاح
أ.د عبد الرحمن علي سعد – د .محمد يحيى عقل 12 -6-2015

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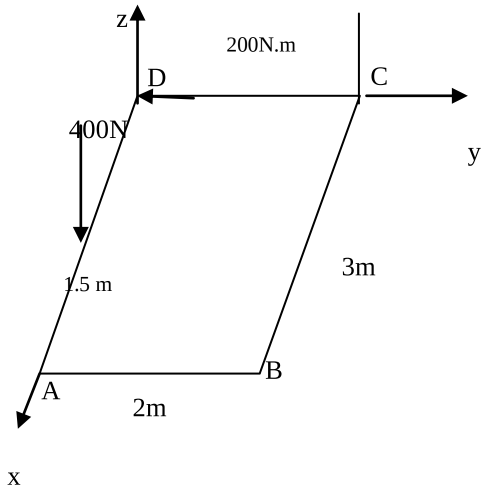

Solution:

$X=0 \rightarrow B_X=0$

$Y=0 \rightarrow B_Y=0$

$Z=0 \rightarrow A_Z+B_Z+T=1400$ N -------(1)

$M_{CB}=0 \rightarrow -A_Z(2)+400(2)+1000(1)=0 \rightarrow A_Z=900$ N

$M_Y=0 \rightarrow -200-A_Z(3)-B_Z(3)+400(1.5)+1000(1.5)=0$

$-200-2700-3B_Z+600+1500=0 \rightarrow B_Z=-266.7$ N

Eq. (1)$\rightarrow$ T=233.333 N

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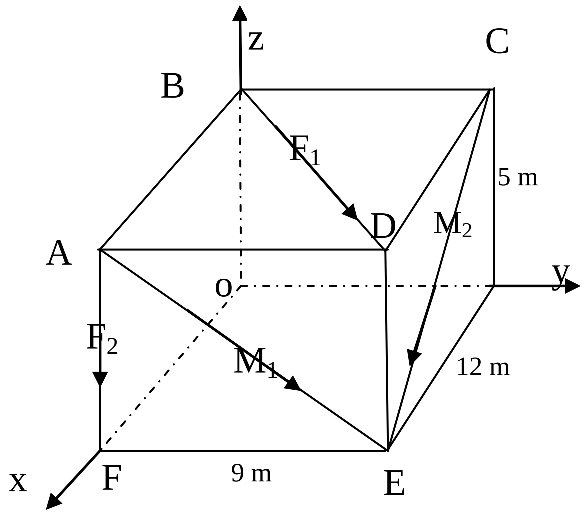
$M_1 = 20\sqrt{106}$ N.m

M_2 = 156 N.m **(10 marks)**



Solution:

$\vec{F}_1=30(12,9,0)\backslash15=(24,18,0)$ N

$\vec{F}_2=(0,0,-20)$ N

$\vec{M}_1=20\sqrt{106}(0,9,-5)\backslash\sqrt{106}=(0,180,-100)$ N.m

$\vec{M}_2=156(12,0,-5)\backslash13=(144,0,-60)$ N.m

$\vec{R}=(24,18,-20)$ N $\rightarrow$ R=36 N

$$\vec{M}_0=(0,180,-100)+(144,0,-60)+\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & 5 \\ 24 & 18 & 0 \end{vmatrix}+\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 12 & 0 & 0 \\ 0 & 0 & -20 \end{vmatrix}$$

$\vec{M}_0=(144,180,-160)+(-90,120,0)+(0,240,0)=(54,540,-160)$ N.m

$$S=\frac{(24,18,-20).(54,540,-160)}{(36)^2}=10.96\ m$$

$$\vec{r}=\frac{1}{1300}\times\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 24 & 18 & -20 \\ 54 & 540 & -160 \end{vmatrix}=\frac{1}{1300}\times(7920,2760,11988)$$

$\vec{r}=(6.09,2.12,9.22)$ m

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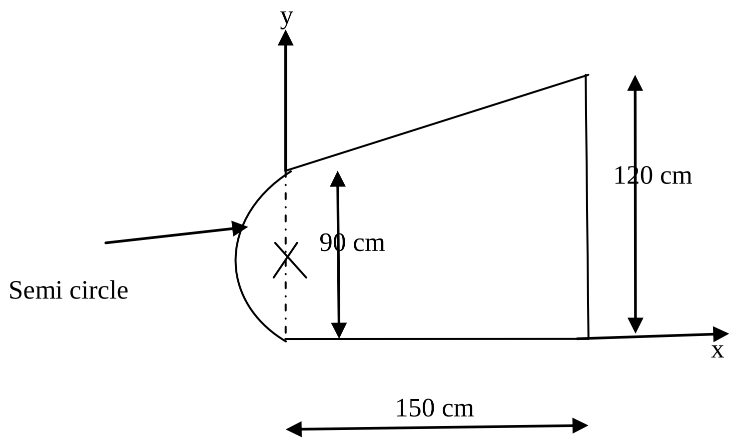
(b) Determine the centroid for the shown area . **(10 marks)**



Solution:

A_1=150*90=13500 cm^2

A_2=0.5*150*30=2250 cm^2

A_3=0.5*3.14*$(45)^2$=3179 cm^2

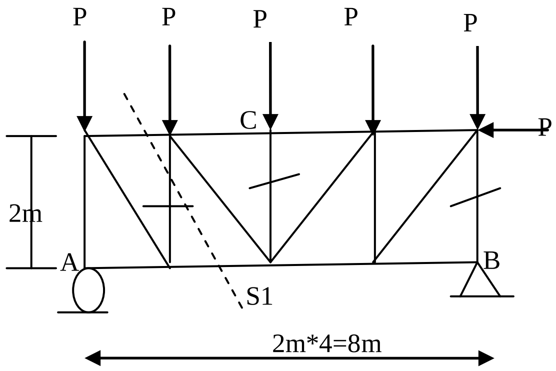
A=18929 cm^2

X=(1\18929)*[13500*75+2250*100-3179*19]=62 cm

Y=(1\18929)*[13500*45+2250*100+3179*45]=52 cm

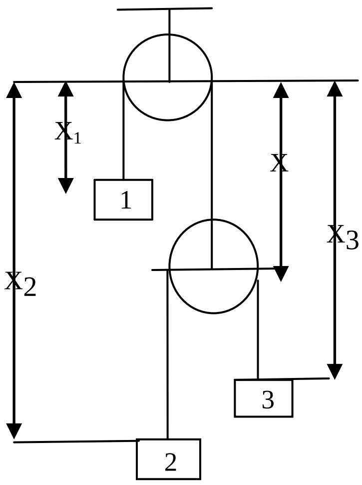
Question No. 3 (20 Marks)[a_1-a_2-b_3]

Determine forces in marked members for the shown truss.
P=100 N



Solution:
$X=0 \rightarrow B_X=100N$
$M_A=0 \rightarrow B_Y(8)-100(2+4+6+8)+100(2)=0 \rightarrow B_Y=225$ N
$Y=0 \rightarrow A_Y+225=500 \rightarrow A_Y=275$ N
Jt (B): $Y=0 \rightarrow F_1=-225$ N (comp.)
Jt (C): $Y=0 \rightarrow F_2=-100$ N (comp.)
Section (S_1): $Y=0 \rightarrow F_3+275-100=0 \rightarrow F_3=175$ N (comp.)

Question No. 4 (20 Marks)[a_1-a_2-c_1]



In the system shown , determine the velocity and acceleration of block (2) at the instant. Knowing that:

$\dot{X}_1 = 6\ m\backslash sec.\uparrow$, $\ddot{X}_1 = 2\ m\backslash sec.^2\downarrow$

$\dot{X}_3 = 3\ m\backslash sec.\uparrow$, $\ddot{X}_3 = 4\ m\backslash sec.^2\downarrow$

Solution

X + X_1=constant$\rightarrow$ $\dot{X} + \dot{X}_1 = 0$,$\ddot{X} + \ddot{X}_1 = 0$

$\dot{X} = -6\ m\backslash sec.$,$\ddot{X} = 2\ m\backslash sec^2$

X_2 –X+X_3-X=constant$\rightarrow$$X_2$+$X_3$-2X=constant

$\dot{X}_2 + \dot{X}_3 - 2\dot{X} = 0$, $\ddot{X}_2 + \ddot{X}_3 - 2\ddot{X} = 0$

At this instant:

$\dot{X}_2 + 3 - 2(-6) = 0$, $\ddot{X}_2 + (-4) - 2(2) = 0$

$\dot{X}_2 = -15\ m\backslash sec$, $\ddot{X}_2 = 8\ m\backslash sec^2$

Question No. 5 (20 Marks)[a_1-b_2-c_1-d_1]

A light elastic string of natural length (L) has one extremity fixed at point (A) and the other end is attached to a stone , the weight of which in equilibrium position would extend the string to a length (L_1). Shoe that if the stone be dropped from rest at A , it will come to instantaneous rest at a depth $\sqrt{L_1{}^2 - L^2}$ below the equilibrium position and this depth attained in time

$$\sqrt{\frac{2L}{g}} + \sqrt{\frac{L_1 - L}{g}}\left[\pi - \cos^{-1}\left(\frac{L_1 - L}{L_1 + L}\right)\right]$$

Solution:

In the static position:

$T_0 = \frac{\lambda}{L}(L_1 - L) = mg \rightarrow \lambda = \frac{mg}{L_1 - L}L$

Motion from a→b

$v_b^2 = v_A^2 + 2gh = 2gL \ \rightarrow \ v_b = \sqrt{2gL}$

$v_b = v_A + gt \rightarrow \sqrt{2gL} = gt \rightarrow t_{AB} = \sqrt{\frac{2L}{g}}$

Motion from b→c

Mg-T=m$\ddot{X}$

$-\frac{\lambda}{L}[X + L_1 - L] + mg = m\ddot{X}$

$-\frac{\lambda}{L}X - mg + mg = m\ddot{X}$

$-\frac{\lambda}{L}X = m\ddot{X}$

$-\frac{mg}{L_1 - L}X = m\ddot{X}$

$\ddot{X} = -\frac{g}{L_1 - L}X = -\omega^2 X$

$\omega = \sqrt{\frac{g}{L_1 - L}}$

At b: $v^2 = \omega^2[A^2 - X^2]$

$2gL = \frac{g}{L_1 - L}[A^2 - (L_1 - L)^2]$

$2LL_1 - 2L^2 = A^2 - L_1^2 + 2LL_1 - L^2$

$A = \sqrt{L_1^2 - L^2}$

$\Theta = \omega t$

$$\pi - \varphi = \sqrt{\frac{g}{L_1 - L}}\ t$$

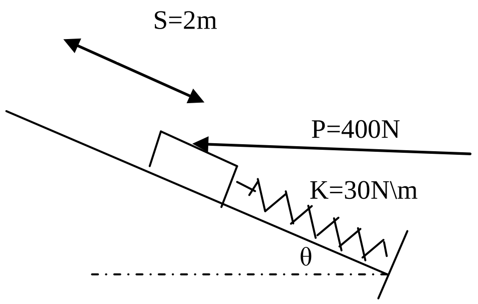
$$\pi - \cos^{-1}\frac{(L_1 - L)}{(L_1 + L)} = \sqrt{\frac{g}{L_1 - L}}\, t$$

$$t_{BC} = \sqrt{\frac{L_1 - L}{g}}\ [\pi - \cos^{-1}\frac{(L_1 - L)}{(L_1 + L)}]$$

$$T = \sqrt{\frac{2L}{g}} + \sqrt{\frac{L_1 - L}{g}}\ [\pi - \cos^{-1}\frac{(L_1 - L)}{(L_1 + L)}]$$

Question No. 6 (20 Marks)[a_1-a_2-b_3-c_1-d_1]

(a) The 10 kg block in the figure rests on the smooth inclined plane . If the spring is originally stretched 0.5 m . Determine the total work done by all the forces acting on the block When a horizontal force P=400 N pushes the block up the plane S=2m , (K=30N\m) , θ=30^0



Solution:

U_w= -10*9.81*2*sin30 = - 98.1 joule
U_p = 400*2*cos30 = 692.8 joule
U_s = 0.5*30*[(0.5)^2 –(2.5)^2] = -90 joule
U_T = -98.1 +692.8 – 90 = 504.7 joule

(b) A square table ABCD whose side is (a) , has raised edges. A particle of elasticity (e) is projected from a point (p) on A\B and hits the sides AB, BC, CD and DA at Q, R and S . Prove that PQ and RS are parallel. If (α) be the angle QPB and BP=x , prve that if the particle returns to P, then:
X(1-e)=α(1-e cot α) .

Solution:

At Q: $\cot\beta = e\cot\alpha$

At R: $\cot\gamma = e\cot(90-\beta) = e\tan\beta = \tan\alpha \rightarrow \gamma = 90-\alpha \rightarrow PQ \backslash\backslash RS$

At S: $\cot\delta = e\cot(90-\gamma) = e\tan\gamma = e\cot\alpha$

$$\tan\alpha = \frac{QB}{BP} = \frac{QB}{x} \rightarrow QB = x\tan\alpha$$

$$\tan\beta = \frac{QC}{RC} = \frac{a-QB}{RC} \rightarrow RC = (a-QB)\cot\beta$$

$$RC = (a - x\tan\alpha)e\cot\alpha = ae\cot\alpha - xe$$

$$\tan\gamma = \frac{RD}{DS} = \frac{a-RC}{DS} \rightarrow DS = (a-RC)\cot\gamma$$

$$DS = (a - ae\cot\alpha + xe)\tan\alpha = a\tan\alpha - ae + xe\tan\alpha$$

$$\tan\delta = \frac{DA}{AP} \rightarrow \frac{1}{e\cot\alpha} = \frac{a-DS}{a-X} = \frac{a - a\tan\alpha + ae - xe\tan\alpha}{a-x}$$

$$a - x = ae\cot\alpha - ae + ae^2\cot\alpha - xe^2$$

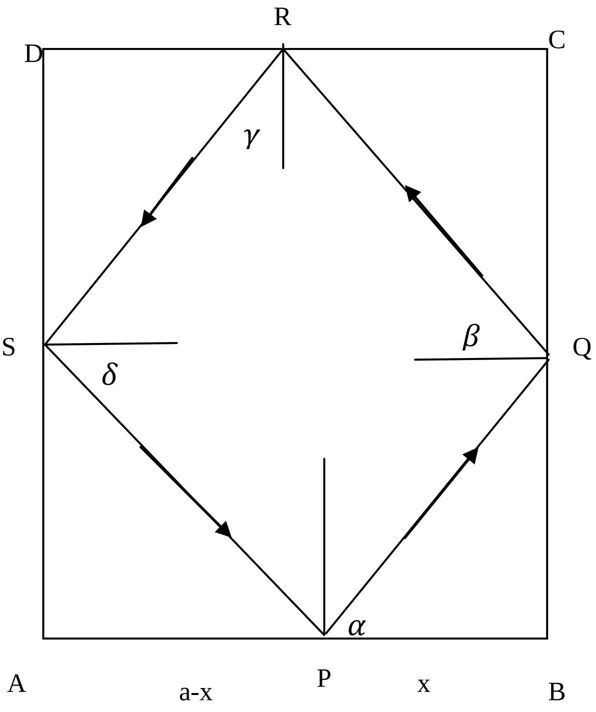
$$xe^2 - x = ae\cot\alpha - ae + ae^2\cot\alpha - a$$

$$x(e^2-1) = ae\cot\alpha(1+e) - a(1+e)$$

$$x(e-1)(e+1) = (1+e)(ae\cot\alpha - a)$$

$$x(e-1) = a(e\cot\alpha - 1)$$

$$x(1-e) = a(1-e\cot\alpha)$$

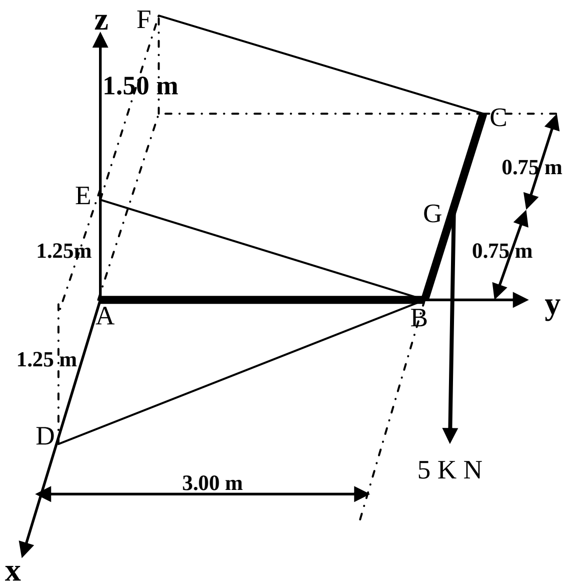


Benha University Faculty of Engineering- Shoubra Mathematics and Engineering Physics Department Year: Preparatory	Final 2nd Term Exam Date: 12 / 6 / 2016 Titel:Mechanics Code: EMP 012 Duration : 4 hours

- Answer all the following questions
- Illustrate your answers with sketches when necessary.
- The exam. consists of three pages
- No. of Questions:…6..
- Total Mark: 120…. Marks

Question No. 1 (20 Marks) [a_1-a_2-b_2]

The light rigid L - shaped member ABC is supported by a ball and –socket joint at A and by three cables. Determined the tension in each cable and the reaction at A caused by the 5 KN load applied at G



$$\vec{T_1} = T_1 \frac{(0,-3,1.25)}{3.25}$$

$$\vec{T_2} = T_2 \frac{(0,-3,1.25)}{3.25}$$

$$\vec{T_3} = T_3 \frac{(1.25,-3,0)}{3.25}$$

$$\overrightarrow{W} = (0,0,-5)$$

$$\vec{R} = \vec{0} \Rightarrow x_A + \frac{1.25}{3.25}T_3 = 0$$

$$y_A - \frac{3}{3.25}T_1 - \frac{3}{3.25}T_2 - \frac{3}{3.25}T_3 = 0$$

$$z_A + \frac{1.25}{3.25}T_1 + \frac{1.25}{3.25}T_2 + 0 = 0$$

$$M_y = 0 \Rightarrow \overrightarrow{M_B} \bullet (-j) = 0$$

$$(\overrightarrow{BC} \times \vec{T_2} + \overrightarrow{BG} \times \overrightarrow{W}) \bullet (\widehat{j}) = 0$$

$$(\begin{vmatrix} \hat{i} & \widehat{j} & \widehat{k} \\ -1.5 & 0 & 0 \\ 0 & -\frac{3}{3.25}T_2 & \frac{1.25}{3.25}T_2 \end{vmatrix} + \left\|\begin{matrix} \hat{i} & \widehat{j} & \widehat{k} \\ -0.75 & 0 & 0 \\ 0 & 0 & -5 \end{matrix}\right\|) \bullet (\widehat{j}) = 0$$

$$[(0, \frac{1.875}{3.25}T_2, \frac{1.875}{3.25}T_2) + (0,-3.75,0)] \bullet (\widehat{j}) = 0$$

$$\frac{1.875}{3.25}T_2 - 3.75 = 0 \Rightarrow T_2 = 6.5KN$$

$$M_x = 0 \Rightarrow \overrightarrow{M_D} \bullet \hat{i} = 0$$

$$\overrightarrow{M_D} = \overrightarrow{DB} \times \vec{T_1} + \overrightarrow{DC} \times \vec{T_2} + \overrightarrow{DG} \times \overrightarrow{W}$$

$$= \begin{vmatrix} \hat{i} & \widehat{j} & \widehat{k} \\ -1.25 & 3 & 0 \\ 0 & -\frac{3}{3.25}T_1 & \frac{1.25}{3.25}T_1 \end{vmatrix} + \begin{vmatrix} \hat{i} & \widehat{j} & \widehat{k} \\ -4.25 & 3 & 0 \\ 0 & -6 & 2.5 \end{vmatrix} + \begin{vmatrix} \hat{i} & \widehat{j} & \widehat{k} \\ -2 & 3 & 0 \\ 0 & 0 & -5 \end{vmatrix}$$

$$= (\frac{3.75}{3.25}T_1, \frac{1.5626}{3.25}T_1, \frac{3.75}{3.25}T_1) + (7.5, 10.625, 25.5) + (-15,-10,0)$$

$$\frac{3.75}{3.25}T_1 + 7.5 - 15 = 0 \Rightarrow \frac{3.75}{3.25}T_1 = 7.5 \Rightarrow T_1 = 6.5KN$$

$$M_z = 0 \Rightarrow \overrightarrow{M_A} \bullet \widehat{k} = 0$$

$$\overrightarrow{M_A} = \overrightarrow{AB} \times \vec{T_1} + \overrightarrow{AC} \times \vec{T_2} + \overrightarrow{AD} \times \vec{T_3} + \overrightarrow{AG} \times \overrightarrow{W}$$

$$= \begin{vmatrix} \hat{i} & \widehat{j} & \widehat{k} \\ 0 & 3 & 0 \\ 0 & -6 & 2.5 \end{vmatrix} + \begin{vmatrix} \hat{i} & \widehat{j} & \widehat{k} \\ -1.5 & 3 & 0 \\ 0 & -6 & 2.5 \end{vmatrix} + \begin{vmatrix} \hat{i} & \widehat{j} & \widehat{k} \\ 1.25 & 0 & 0 \\ \frac{1.25}{3.25} & -\frac{3}{3.25} & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \widehat{j} & \widehat{k} \\ -0.75 & 0 & 0 \\ 0 & 0 & -5 \end{vmatrix}$$

$$0 + 9 - \frac{3.75}{3.25}T_3 + 0 = 0 \Rightarrow T_3 = 7.8KN$$

$$X_A = -19.5KN, Y_A = 19.2KN, Z_A = -5KN$$

Question No. 2 (20 Marks)[a_1-b_1]

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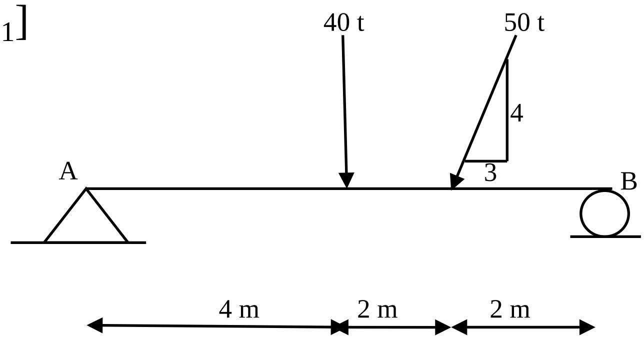
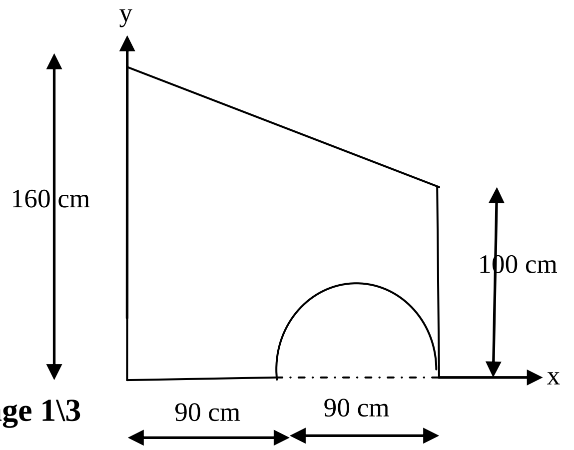


(a) Determine the reactions at the supports for the shown beam . The radius of friction circle at the hinge A is 10 cm .

(10 marks)

$$X = 0 \Rightarrow X_A = 30t$$
$$Y = 0 \Rightarrow Y_A + Y_B = 80t$$
$$M_A = 0 \Rightarrow M_f + Y_B(8) - 160 - 240 = 0$$
$$M_f + 8Y_B = 400$$
$$M_f = 0.1R_A = 0.1\sqrt{900 + Y_A^2}$$
$$0.1\sqrt{900 + Y_A^2} + 8(80 - Y_A) = 400$$
$$0.1\sqrt{900 + Y_A^2} + 640 - 8Y_A = 400$$
$$0.1\sqrt{900 + Y_A^2} = -240 + 8Y_A$$
$$\sqrt{900 + Y_A^2} = -2400 + 80Y_A$$
$$900 + Y_A^2 = 5760000 - 384000Y_A + 6400Y_A^2$$
$$6399Y_A^2 - 384000Y_A + 5759100 = 0$$
$$Y_A^2 - 60.009Y_A + 900 = 0$$
$$Y_A = \frac{60 + \sqrt{3601.1 - 4(900)}}{2}$$
$$Y_A = 30.5t, Y_B = 49.5t, M_f = 42.78m.t$$

(b) Determine the centroid for the shown area . **(10 marks)**



Page 1\3

$$A_1 = 180 \times 100 = 18000 cm^2$$

$$A_2 = \frac{1}{2} \times 180 \times 60 = 5400 cm^2$$

$$A_3 = \frac{1}{2} \times \pi \times (45)^2 = 3179 cm^2$$

$$A = 18000 + 5400 - 3179 = 20221 cm^2$$

$$\bar{x} = \frac{1}{20221}[18000 \times 90 + 5400 \times 60 - 3179 \times 135]$$

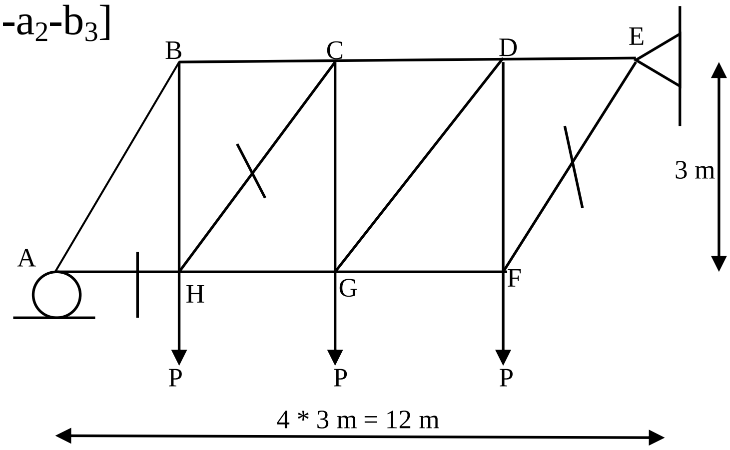
$$= \frac{1}{20221}[1620000 + 324000 - 429165] = 74.9 cm$$

$$\bar{y} = \frac{1}{20221}[18000 \times 50 + 5400 \times 120 - 3179 \times 19]$$

$$= \frac{1}{20221}[900000 + 648000 - 60401] = 73.6 cm$$

Question No. 3 (20 Marks)[a_1-a_2-b_3]

Determine forces in marked members for the shown truss.
P= 5 t



$$X = 0 \Rightarrow X_E = 0$$
$$M_A = 0 \Rightarrow Y_E(12) - 5(3) - 5(6) - 5(9) = 0$$
$$12Y_E = 15 + 30 + 45 = 90$$
$$Y_E = 7.5t \Rightarrow Y_A = 7.5t$$
$$Jt(A):$$
$$F_{AH} = 0$$
$$Jt(E):$$
$$Y_E - F_{EF} \times \frac{1}{\sqrt{2}} = 0 \Rightarrow F_{EF} = 7.5\sqrt{2}t(tens.)$$
$$\sec.s - s, left:$$
$$Y = 0 \Rightarrow Y_A - 5 + T_{HC} \times \frac{1}{\sqrt{2}} = 0 \Rightarrow T_{HC} = -5\sqrt{2}t(comp)$$

Question No. 4 (20 Marks)[a_1-a_2-c_1]

(a) A particle is projected from a certain point. It is noticed that its range on the horizontal plane which passes through the point of projection is equal to three times the maximum height above the point of projection , and its velocity after two seconds from the projection is equal to velocity of projection , find the velocity of projection and find also the position of the particle after 5 seconds from the beginning of motion .

$$R = 3h$$

$$\frac{v_0{}^2 \sin 2\alpha}{g} = 3\frac{v_0{}^2 \sin^2\alpha}{2g}$$

$$2\sin\alpha\cos\alpha = \frac{3}{2}\sin^2\alpha$$

$$\tan\alpha = \frac{4}{3}$$

$$T = \frac{2v_0\sin\alpha}{g} = 2$$

$$v_0(\frac{4}{5}) = 32$$

$$v_0 = 40\, ft \backslash \sec$$

$$x = tv_0\cos\alpha$$

$$when : t = 5$$

$$x = 5(40)(\frac{3}{5}) = 120\, ft$$

$$y = tv_o\sin\alpha - \frac{1}{2}gt^2$$

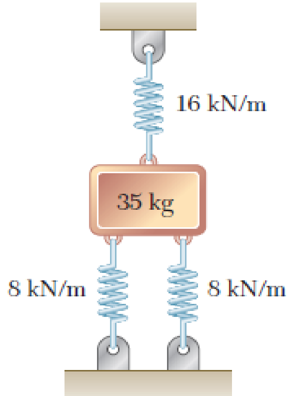
$$when : t = 5$$

$$y = 5(40)(\frac{4}{5}) - \frac{1}{2}(32)(5)^2 = -240\, ft$$

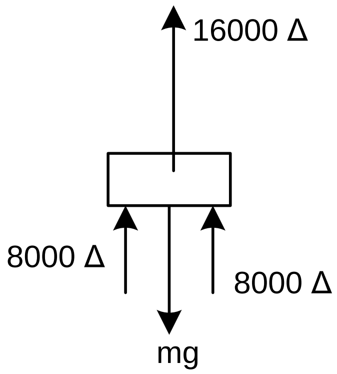
(b)A 35-Kg block is supported by the spring arrangement shown. The block is moved vertically downward from its equilibrium position and released. Knowing that the amplitude of the resulting motion is 4.5 cm. knowing that (g= 9.81 m/s^2).

Determine:

(a) Vertical displacement in each spring.
(b) Prove that the motion is simple harmonic motion
(c) The period and frequency of the motion
(d) The maximum velocity and maximum acceleration of the block



بيان الجسم الحر عند وضع الاتزان



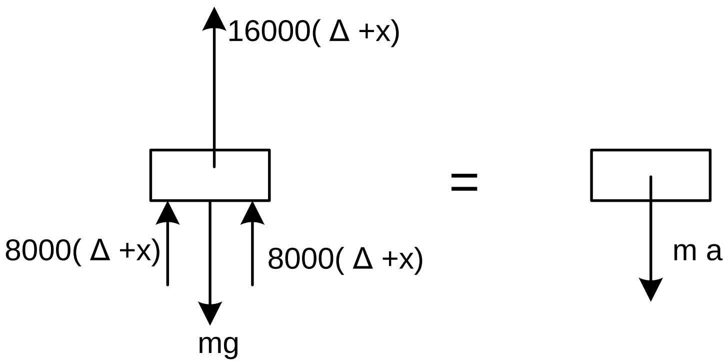
$$\sum f_y = 0$$

$$16000\,\Delta + 8000\,\Delta + 8000\,\Delta = \mathrm{mg}$$

$$\Delta = 0.01073\ m$$

b)

بيان الجسم الحر عند وضع الحركة



$ma = mg - 16000(\Delta + \mathrm{x}) - 8000(\Delta + \mathrm{x}) - 8000(\Delta + \mathrm{x})$

S.H.M (1) $a = -914.285\ x$

c)

$\because a = -\omega^2 x$

Comparing with equation (1)

$\omega^2 = 914.285$

$\omega = 30.2371\ rad/sec$

$\tau = \frac{2\Pi}{\omega} = \frac{2\Pi}{30.2371} sec$

$f = \frac{1}{\tau}$

c)

From equation (1)

$a = -914.285\ x$

So $\frac{vdv}{dx} = -914.285\ x$

Separating variables and integrate both side one can get

$\frac{v^2}{2} = -914.285\ \frac{x^2}{2} + C$

Substituting v=0 and x=4.5 cm get

(2) $v^2 = -914.285\ (4.5^2 - x^2)$

To get the max. Velocity put a=0 get x=0

Substituting x=0 in equation (2) get

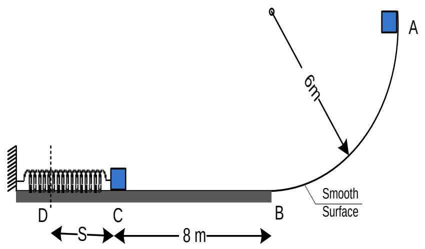
$v_{max} = 136\ cm/sec$

And

$a_{max} = \omega^2\ A = 41.15\ m/sec^2$

Question No. 5 (20 Marks)[a_1-b_2-c_1-d_1]

(a) A particle of weight 98 N start its motion from rest at point A, it moves on the smooth surface from A to B then over the rough surface whose coefficient of kinetic friction is 0.2 from B to C then it compress the spring of stiffness (20 N \ m) till the position D. Knowing that ($g = 9.81\ m/s^2$)
Determine:
(a) The velocity at (B) & (C) .
(b) The maximum compression in the spring at (D).
(c) The reaction of the surface just before and after (B)



a)

بتطبيق مبدأ الشغل وطاقة الحركة بين A ,B

$$\mathrm{W}_{A\to B} = T_B - T_A$$
$$\mathrm{W}_{mg} = T_B - T_A$$
$$mgh = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$
$$98 * 6 = \frac{1}{2}\left(\frac{98}{9.81}\right) v_B^2 - 0$$
$$v_B = 10.85\ m/s$$

بتطبيق مبدأ الشغل وطاقة الحركة بين C ,A

$$\mathrm{W}_{A\to C} = T_C - T_A$$
$$\mathrm{W}_{mg} + \mathrm{W}_{friction} = T_C - T_A$$
$$mgh - \mu N * 8 = \frac{1}{2}mv_C^2 - 0$$
$$98 * 6 - 0.2 * 98 * 8 = \frac{1}{2}\left(\frac{98}{9.81}\right) v_C^2 - 0$$
$$v_C = 9.29\ m/s$$

b)

بتطبيق مبدأ الشغل وطاقة الحركة بين D ,A

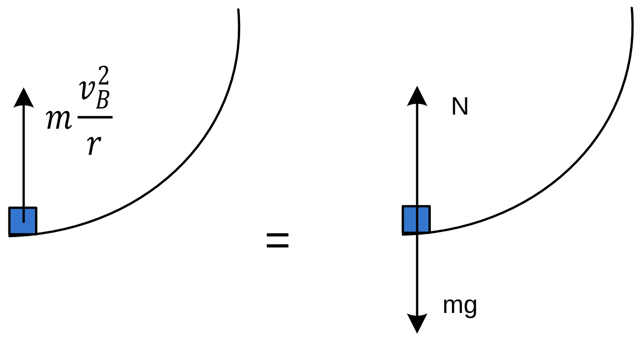
$$\mathrm{W}_{A\to D} = T_C - T_A$$
$$\mathrm{W}_{mg} + \mathrm{W}_{friction} + \mathrm{W}_{spring} = T_D - T_A$$
$$mgh - \mu N * (8 + \mathrm{s}) + \frac{1}{2}K(\mathrm{s}_1^2 - \mathrm{s}_2^2) = 0 - 0$$
$$98 * 6 - 0.2 * 98 * (8 + \mathrm{s}) + \frac{1}{2} * 20 * (0 - \mathrm{s}_2^2) = 0$$

$\mathrm{Sos}_2 = -7.619\ m\ refused\ or\ \mathrm{s}_2 = 5.65\ m$ accepted $\qquad -10\mathrm{s}_2^2 - 19.6\mathrm{s}_2 + 431.2 = 0$

c)

لايجاد رد الفعل قبل وبعد نقطة B وذلك برسم بيان الجسم الحر

- رد الفعل قبل نقطة B

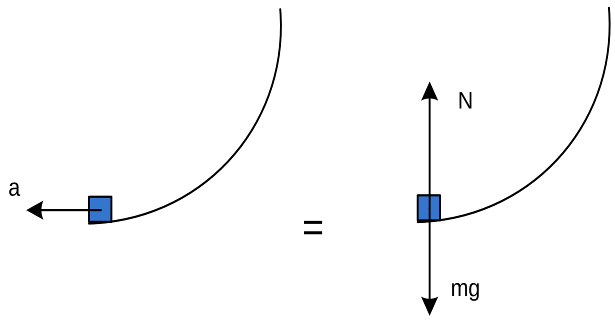


$$m\frac{v_B^2}{r} = N - mg$$

So

$$N = m\frac{v_B^2}{r} + mg = \left(\frac{98}{9.81}\right) * \frac{(10.85)^2}{6} + 98 = 294\ N$$

- رد الفعل بعد نقطة B



$$0 = N - mg$$

So

$$N = mg = 98\ N$$

(b)A particle moves down outside the surface of a smooth vertical circular disk of radius*R*, it starts at a depth $\frac{1}{2}R$ below the highest point, show that it leaves the disk at a height $\frac{2}{3}R$ under the highest point.

$$\frac{1}{2}mv_B{}^2-\frac{1}{2}mv_A{}^2=mg(h-\frac{R}{2})$$

$$v_B{}^2=2g(h-\frac{R}{2})$$

$$at, B:$$

$$\frac{mv_B{}^2}{R}=mg\cos\theta-N$$

$$v_B{}^2=gR\cos\theta$$

$$2g(h-\frac{R}{2})=gR\cos\theta$$

$$2h-R=R\times\frac{R-h}{R}$$

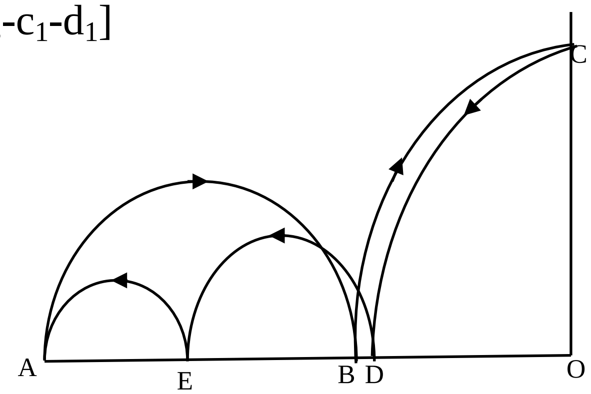
$$2h-R=R-h$$

$$3h=2R$$

$$h=\frac{2}{3}R$$

Question No. 6 (20 Marks)[a_1-a_2-b_3-c_1-d_1]

A ball is projected from a point in horizontal plane, after one rebound from the plane it strikes directly against a vertical wall, after two more rebounds it returns to the point of projection. Prove that
$\boldsymbol{e = 2(1 - e^3)}$.



$$t_1 = \frac{2v}{g} \Rightarrow AB = ut_1 = \frac{2uv}{g}$$

$$t_2 = \frac{ev}{g} \Rightarrow BO = ut_2 = \frac{euv}{g}$$

$$OD = t_2 eu = \frac{e^2uv}{g}$$

$$t_3 = \frac{2e^2v}{g} \Rightarrow DE = t_3 eu = \frac{2e^3uv}{g}$$

$$t_4 = \frac{2e^3v}{g} \Rightarrow EA = t_4 eu = \frac{2e^4uv}{g}$$

$$AB + BO = OD + DE + EA$$

$$\frac{2uv}{g} + \frac{euv}{g} = \frac{e^2uv}{g} + \frac{2e^3uv}{g} + \frac{2e^4uv}{g}$$

$$2 + e = e^2 + 2e^3 + 2e^4$$

$$(1+e)(2e^3 + e - 2) = 0$$

$$2e^3 + e - 2 = 0$$

$$e = 2(1 - e^3)$$

مع التمنيات بالنجاح
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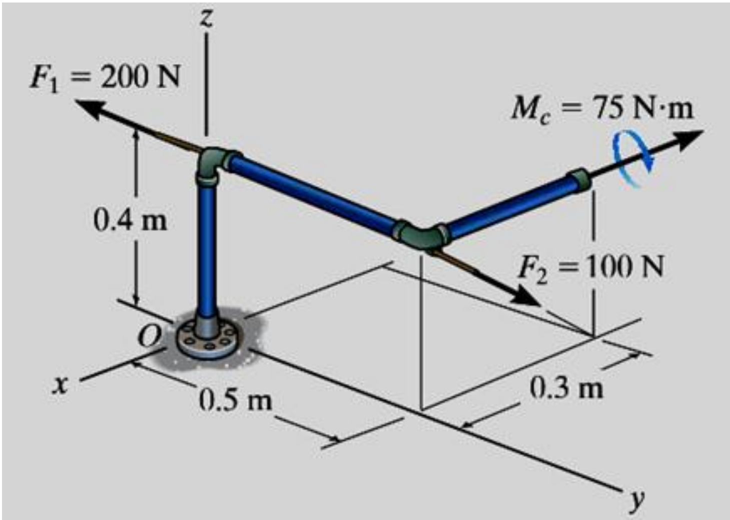
Preparatory Year

Final 2nd Term Exam
Date: 21 / 5 / 2018
Course Title: Mechanics
Course Code: ENP 012
Duration: 4 hours

- Answer all the following questions
- Illustrate your answers with sketches when necessary
- The exam consists of **2** parts
- Total exam marks : **120**
- No. of questions of part **1**: 3
- Part **1** consists of **2** pages

Question No. 1 [20 Marks]

(a) In the figure shown, replace the loading system by an equivalent resultant force and couple moment acting at point O. [8 Marks]



Solution

$$\vec{F}_1 = -200\vec{j}, \quad \vec{F}_2 = -100(\frac{3}{5})\vec{i} - 100(\frac{4}{5})\vec{k}.$$

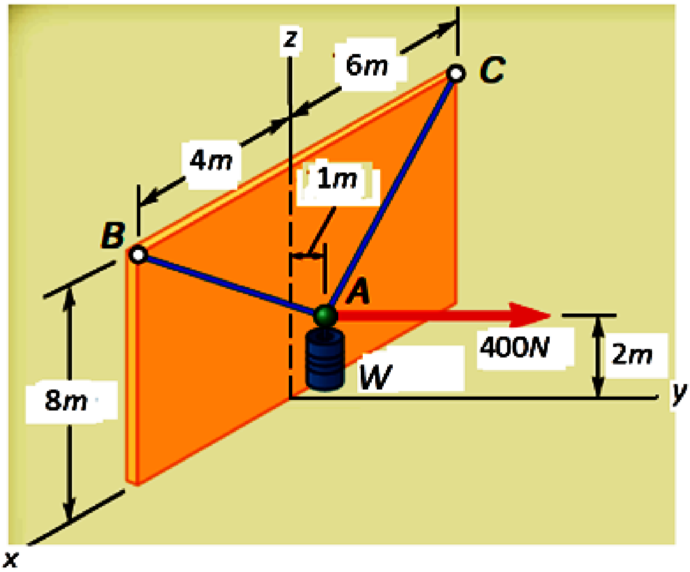
$$\therefore \vec{F}_R = -60\vec{i} - 200\vec{j} - 80\vec{k}.$$

$$\vec{M}_R = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 - 75\vec{i}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 0.4 \\ 0 & -200 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0.5 & 0.4 \\ -60 & 0 & -80 \end{vmatrix} - 75\vec{i}$$

$$= -35\vec{i} - 24\vec{j} + 30\vec{k}.$$

(b) A cylinder of weight W is hung by means of two cables AB and AC, which are attached to the top of a vertical wall. A horizontal force of magnitude 400N perpendicular to the wall holds the cylinder in the position shown in the opposite figure. Determine the magnitude of the weight W and the tension in each cable. [12 Marks]



$$\vec{F}_1 = 400\vec{j}, \quad \vec{W} = -W\,\vec{k},$$

$$\vec{T}_{AC} = \vec{T}_1 = T_1(-6\vec{i} - \vec{j} + 6\vec{k})/\sqrt{73}$$

$$\vec{T}_{AB} = \vec{T}_2 = T_2(4\vec{i} - \vec{j} + 6\vec{k})/\sqrt{53}$$

$$\sum F_x = 0 \Rightarrow (-6/\sqrt{73})T_1 + (4/\sqrt{53})T_2 = 0$$

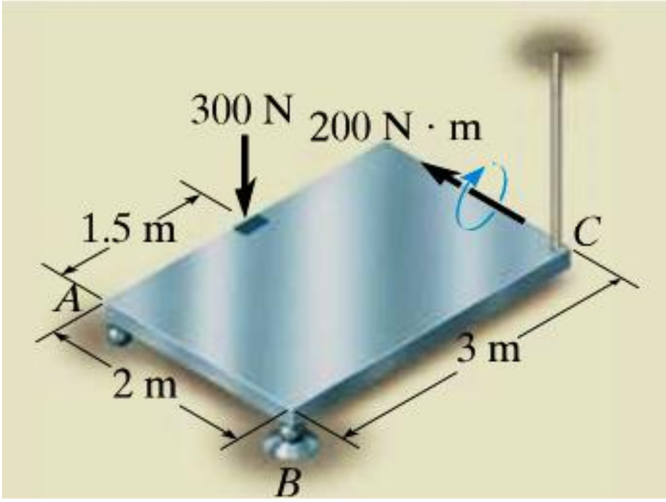
$$\sum F_y = 0 \Rightarrow 400 - (1/\sqrt{73})T_1 - (1/\sqrt{53})T_2 = 0$$

$$\sum F_z = 0 \Rightarrow (6/\sqrt{73})T_1 + (6/\sqrt{53})T_2 - W = 0$$

$$\Rightarrow T_1 = 160\sqrt{73}N, \quad T_2 = 240\sqrt{53}N, \quad W = 2400N$$

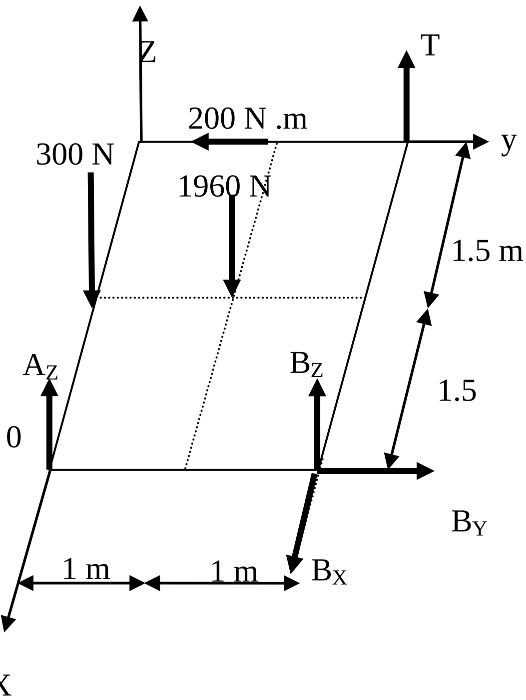
Question No. 2 [25 Marks]

(a) The plate shown in the opposite figure has a mass of 200 Kg and is subjected to a force and couple moment along its edges. If it is supported in the horizontal plane by a roller at A, a ball-and-socket joint at B, and a cord at C, determine the reactions at these supports. [13 Marks]

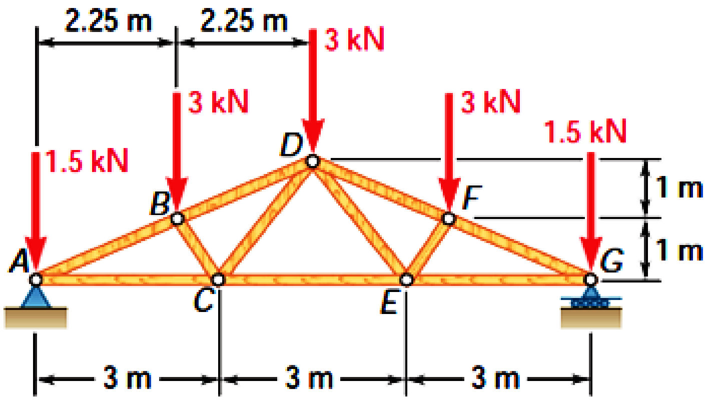


Solution

$$\sum X = 0 \Rightarrow B_X = 0$$
$$\sum Y = 0 \Rightarrow B_Y = 0$$
$$\sum Z = 0 \Rightarrow A_Z + B_Z + T = 2260 \ldots\ldots\ldots(1)$$
$$\sum M_{CB} = 0 \Rightarrow -A_Z(3) + 300(2) + 1960(1) = 0$$
$$A_Z = 1280N$$
$$\sum M_y = 0 \Rightarrow -200 - A_Z(3) - B_Z(3) + 300(1.5) + 1960(1.5) = 0$$
$$-200 - 1280(3) - 3B_Z + 450 + 2940 = 0$$
$$-200 - 3840 - 3B_Z + 3390 = 0$$
$$-4040 - 3B_Z + 3390 = 0$$
$$3B_Z = -650 \Rightarrow B_Z = -217N$$
$$eq.(1) \Rightarrow 1280 - 217 + T = 2260$$
$$T = 1197N$$



(b) In the truss shown in the opposite figure, the web members BC and EF are perpendicular to the inclined members at their midpoints. Use the method of sections to determine the force in members DF, DE, and CE.
[12 Marks]



Solution

$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y + G_y = 12 \quad KN$$

$$\text{Symmetry} \Rightarrow A_y = G_y \Rightarrow G_y = 6 \quad KN$$

$$\sum M_D = 0 \Rightarrow$$

$$-F_{CE} \times 2 - 3 \times 2.25 - 1.5 \times 4.5 + 6 \times 4.5 = 0$$

$$\Rightarrow F_{CE} = 6.75 \quad KN \ (T)$$

$$\sum M_G = 0 \Rightarrow$$

$$-F_{DE} \times (4/5) \times 3 + 3 \times 2.25 = 0$$

$$\Rightarrow F_{DE} = 2.81 \quad KN \ (T)$$

$$\sum M_E = 0 \Rightarrow$$

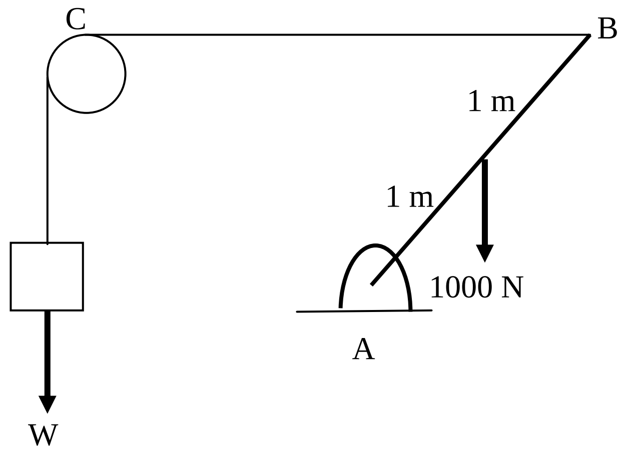
$$(6 - 1.5) \times 3 - 3 \times 0.75$$

$$-F_{DF} \times \frac{1}{\sqrt{1+(2.25)^2}} \times 0.75 - F_{DF} \times \frac{2.25}{\sqrt{1+(2.25)^2}} \times 1 = 0$$

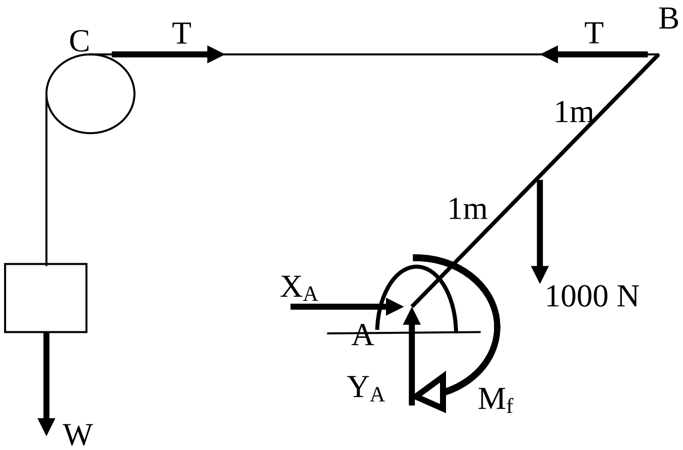
$$\Rightarrow F_{DF} = 9.23 \quad KN \ (C)$$

Question No. 3 [15 Marks]

A uniform rod AB of weight 1000 N and length 2 m is hinged at A at a rough hinge, the radius of its friction circle is r_f = 20 cm, inclines at an angle 45^0 to the horizontal in the equilibrium position. A horizontal rope CB tied it at B, this rope passes on a rough cylinder fixed at C and its coefficient of friction = $1/\pi$. A weight W is fixed at the free end of the rope. Determine the two limiting values of W.



Solution



$1) W \downarrow$

$Cylinder(C):$

$$W = Te^{\mu\theta} = T^{\frac{1}{\pi}\times\frac{\pi}{2}} = Te^{\frac{1}{2}} \Rightarrow T = We^{-\frac{1}{2}}$$

$Rod(AB):$

$$\sum Y = 0 \Rightarrow Y_A = 1000N$$

$$\sum X = 0 \Rightarrow X_A - T = 0 \Rightarrow X_A = We^{-\frac{1}{2}}$$

$$X_A \approx 0.61W = T$$

$$R_A = \sqrt{X_A{}^2 + Y_A{}^2} = \sqrt{(0.61W)^2 + (1000)^2}$$

$$M_f = R_A \times r_f = 0.2\sqrt{(0.61W)^2 + (1000)^2}$$

$$\sum M_A = 0 \Rightarrow -1000(1\cos 45^0) + T(2\sin 45^0) - M_f = 0$$

$$-710 + 0.86W - 0.2\sqrt{(0.61W)^2 + (1000)^2} = 0$$

$$0.2\sqrt{(0.61W)^2 + (1000)^2} = 0.86W - 71$$

$$W = 1110N, or, W = 580N\ (refused)$$

$2) W \uparrow$

$Cylinder(C):$

$$T = We^{\mu\theta} = W^{\frac{1}{\pi}\times\frac{\pi}{2}} = We^{\frac{1}{2}} \Rightarrow T = We^{\frac{1}{2}}$$

$Rod(AB):$

$$\sum Y = 0 \Rightarrow Y_A = 1000N$$

$$\sum X = 0 \Rightarrow X_A - T = 0 \Rightarrow X_A = We^{\frac{1}{2}}$$

$$X_A \approx 1.65W = T$$

$$R_A = \sqrt{X_A{}^2 + Y_A{}^2} = \sqrt{(1.65W)^2 + (1000)^2}$$

$$M_f = R_A \times r_f = 0.2\sqrt{(1.65W)^2 + (1000)^2}$$

$$\sum M_A = 0 \Rightarrow -1000(1\cos 45^0) + T(2\sin 45^0) - M_f = 0$$

$$-710 + 2.33W - 0.2\sqrt{(1.65W)^2 + (1000)^2} = 0$$

$$0.2\sqrt{(1.65W)^2 + (1000)^2} = 2.33W - 71$$

$$W = 210N, or, W = 410N\ (refused)$$

Question No. 4 [20 Marks]

(a) If the radial and transversal components of the velocity of a particle are $a r^2$, $2 a r^2$, where a is a constant . Find the radial and transversal components of acceleration , find also the equation of the path , knowing that $r = a$ when $\theta = 0$. [10 Marks]

$$v_r = ar^2 \Rightarrow \dot{r} = ar^2$$

$$v_\theta = 2ar^2 \Rightarrow r\dot{\theta} = 2ar^2 \Rightarrow \dot{\theta} = 2ar$$

$$\ddot{r} = 2ar\dot{r} = 2ar(ar^2) = 2a^2r^3$$

$$\ddot{\theta} = 2a\dot{r} = 2a(ar^2) = 2a^2r^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 2a^2r^3 - r(2ar)^2 = 2a^2r^3 - 4a^2r^3 = -2a^2r^3$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r(2a^2r^2) + 2(ar^2)(2ar) = 2a^2r^3 + 4a^2r^3 = 6a^2r^3$$

$$\frac{v_r}{v_\theta} = \frac{\dot{r}}{r\dot{\theta}} = \frac{ar^2}{2ar^2} = \frac{\frac{dr}{dt}}{\frac{rd\theta}{dt}} \rightarrow \frac{dr}{rd\theta} = \frac{1}{2} \rightarrow \frac{dr}{r} = \frac{1}{2}d\theta$$

$$\int \frac{dr}{r} = \frac{1}{2}\int d\theta \rightarrow \ln r = \frac{1}{2}\theta + c$$

$$when, r = a, \theta = 0 \rightarrow \ln a = 0 + c \rightarrow c = \ln a$$

$$\ln r = \frac{1}{2}\theta + \ln a \rightarrow \ln r - \ln a = \frac{1}{2}\theta \rightarrow \ln\frac{r}{a} = \frac{1}{2}\theta$$

$$\frac{r}{a} = e^{\frac{1}{2}\theta} \rightarrow r = ae^{\frac{1}{2}\theta}$$

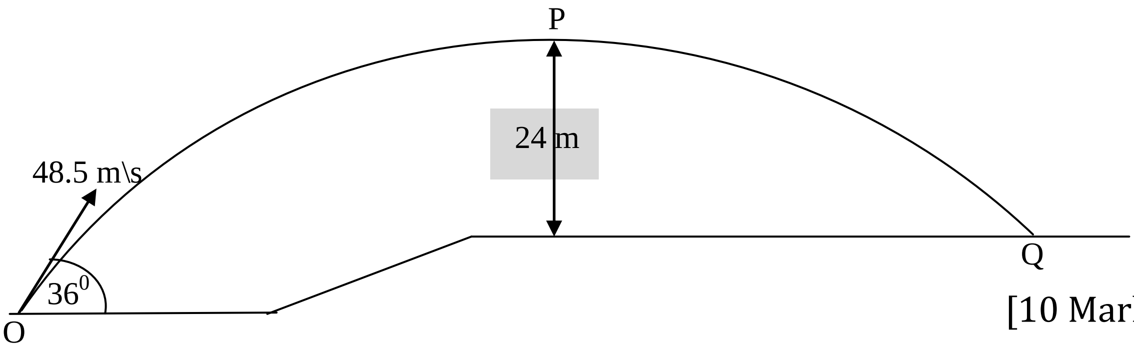
(b)A golf ball at point O is given an initial velocity 48.5 m\s at an angle 36^0 to the horizontal . The ball reaches the maximum height at point P and then continues to hit the ground at point Q.

a) Find the time from O to Q

b) Calculate the horizontal distance (x_Q) travelled by the ball

c) Find the velocity direction at point O

d) Find the time from the beginning of motion for which the ball will take orthogonal direction to the initial projectile direction at O



[10 Marks]

T_Q :

$$h=\frac{V_0^2\sin^2\alpha}{2g}=\frac{(48.5)^2(\sin 36^0)^2}{2\times 9.8}=41.5m \Rightarrow h_Q=h-24=41.5-24=17.5m$$

$$y=V_0\sin\alpha t-\frac{gt^2}{2}\Rightarrow 17.5=(48.5)(\sin 36^0)t-\frac{9.8}{2}t^2$$

$$4.9t^2-28.5t+17.5=0t=5.1\sec.,or,t=0.7\sec(refused)$$

X_Q :

$$x=tV_0\cos\alpha=5.1\times 48.5\times\cos 36^0=200m$$

Θ_Q :

$$\dot{x}=v_0\cos\alpha=48.5\times\cos 36^0=39.2m$$

$$\dot{y}=v_0\sin\alpha-gt=48.5(\sin 36^0)-9.8(5.1)=-21.5m\setminus s$$

$$\tan\theta=\frac{\dot{y}}{\dot{x}}=\frac{21.5}{39.2}\Rightarrow\theta=28.7^0\Rightarrow\theta_Q=360-28.7=331.3^0$$

T (orthogonal) :

$$\dot{x}=39.2m\setminus s,\dot{y}=v_0\sin\alpha-gt=48.5\sin 36^0-9.8t=28.5-9.8t$$

$$\theta_\perp=90+(90-36)=144^0$$

$$\tan\theta_\perp=\frac{\dot{y}}{\dot{x}}\Rightarrow\tan 144^0=\frac{28.5-9.8t}{39.2}\Rightarrow t_\perp=5.8\sec$$

Question No. 5   [20 Marks]

(a)Find the work done by a variable force $\vec{F} = x\vec{i} - 2xy\vec{j}$ to displace a particle from the point O (0 , 0) to the point B (3 , 6) on the broken line path O A B , where A (3 , 0) . [10 Marks]

$$\vec{F} = x\vec{i} - 2xy\vec{j}$$

$$W_{O\to A} = \int_{O}^{A} \vec{F} \bullet dx\vec{i} = \int_{0}^{3} x dx = 4.5$$

$$W_{A\to B} = \int_{A}^{B} \vec{F} \bullet dy\vec{i} = \int_{0}^{3} -2xydy = \int_{0}^{3} -2(3)ydy = -108$$

$$W_{O\to B} = W_{O\to A} + W_{A\to B} = 4.5 - 108 = -103.5 U.of.W$$

(b)A particle of mass (m) is fixed at one end of an inelastic string of length (a) , the other end of the string is fixed at a point O . The particle is projected horizontally with an initial velocity $\sqrt{\frac{7ag}{2}}$. Show the tension is vanished at an angle 120^0 , when the string becomes slack the particle will move as a free projectile . Find the maximum height up the point of projection .

[10 Marks]

$$A \to B: \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = -mg(a + a\cos\theta)$$

$$v_B^2 = v_A^2 - 2ga\cos\theta............(1)$$

$At(B):$

$$mg\cos\theta + T_B = \frac{mv_B^2}{a} \Rightarrow v_B^2 = ga\cos\theta.......(2)$$

$$ga\cos\theta = \frac{7}{2}ga - 2ga\cos\theta \Rightarrow \cos\theta = \frac{1}{2}$$

$$\theta = 60^0 \Rightarrow \theta_B = 180 - 60 = 120^0$$

$\Pr oj.:$

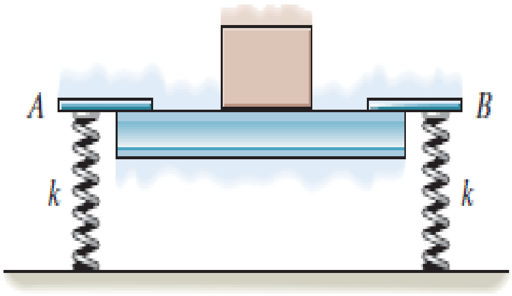
$$v_0 = v_B = \sqrt{\frac{ga}{2}}, \alpha = \theta = 60^0$$

$$h = \frac{v_0^2 \sin^2\alpha}{2g} = \frac{(\frac{ga}{2})(\sin 60^0)^2}{2g} = \frac{3}{16}a$$

$$h_A = a + h = 1.5a + \frac{3}{16}a = \frac{27}{16}a$$

Question No. 6   [20 Marks]

1)The uniform beam is supported at its ends by two springs A and B, each having the same stiffness k. When nothing is supported on the beam, it has a period of vertical vibration of 0.83 s. If a 50-kg mass is placed at its center, the period of vertical vibration is 1.52 s. Compute the stiffness of each spring and the mass of the beam. [10 Marks]



Solution

$$\because \tau = 2\Pi\sqrt{\frac{m}{k}}$$

$$\therefore \frac{\tau^2}{(2\Pi)^2} = \frac{m}{k} \qquad (1)$$

$$\therefore \frac{\tau^2}{(2\Pi)^2} = \frac{m}{k}$$

For t=0.83 and mass = m the following relation can be obtained as follows:

$$\frac{(0.83)^2}{(2\Pi)^2} = \frac{m}{2k} \qquad (2)$$

For t=1.52 sec. and mass = m+50 the following relation can be obtained as follows:

$$\frac{(1.52)^2}{(2\Pi)^2} = \frac{m+50}{2k} \qquad (3)$$

By dividing equation (3) by equation (2) the following equation obtained as follows:

$$m + 50 = 3.35375m \qquad (4)$$

So

$$2.35375m = 50$$
$$m = 21.2427\ Kg \qquad (5)$$

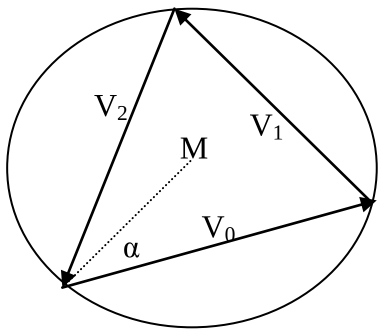
Subsisting equation (5) into equation (2) one can get:

$$k = 608.671\ N/m$$

(B) On a circular table of raised edges , a particle is projected with velocity V_0 in a direction makes an angle α with the radius as shown . If the particle impinges the edge of the table two times and then returns to the point of projection , prove that :

$$\tan^2\alpha = \frac{e^3}{1+e+e^2}$$

$$V_2 = V_0 e^{\frac{3}{2}}$$

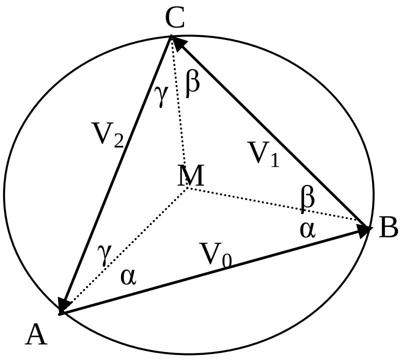


Where : e is the modulus of restitution

V_2 : is the velocity of the particle when returns back

To the point of projection

Ans. :



$$At(B): V_1\sin\beta = V_0\sin\alpha, V_1\cos\beta = eV_0\cos\alpha$$
$$At(C): V_2\sin\gamma = V_1\sin\beta, V_2\cos\gamma = eV_1\cos\beta$$
$$\tan\beta = \frac{1}{e}\tan\alpha, \tan\gamma = \frac{1}{e}\tan\beta = \frac{1}{e^2}\tan\alpha$$
$$2\alpha+2\beta+2\gamma = 180^0 \Rightarrow \alpha+\beta+\gamma = 90^0 \Rightarrow \beta+\gamma = 90^0-\alpha$$
$$\tan(\beta+\gamma) = \tan(90^0-\alpha)$$
$$\frac{\tan\beta+\tan\gamma}{1-\tan\beta\tan\gamma} = \cot\alpha \Rightarrow \frac{\frac{1}{e}\tan\alpha+\frac{1}{e^2}\tan\alpha}{1-\frac{1}{e}\tan\alpha\times\frac{1}{e^2}\tan\alpha} = \frac{1}{\tan\alpha}$$
$$\frac{e^2\tan\alpha+e\tan\alpha}{e^3-\tan^2\alpha} = \frac{1}{\tan\alpha} \Rightarrow \tan^2\alpha = \frac{e^3}{1+e+e^2}$$
$$V_2^2 = V_1^2\sin^2\beta + e^2V_1^2\cos^2\beta$$
$$V_2^2 = V_0^2\sin^2\alpha + e^4V_0^2\cos^2\alpha$$
$$V_2^2 = V_0^2(\sin^2\alpha+e^4\cos^2\alpha) = V_0^2\left(\frac{e^3}{1+e+e^2+e^3}+\frac{e^3(1+e+e^2)}{1+e+e^2+e^3}\right)$$
$$V_2^2 = V_0^2e^3 \Rightarrow V_2 = V_0 e^{\frac{3}{2}}$$

Benha University
Faculty of Engineering (Shoubra)
Engineering Mathematics and Physics Department
Preparatory Year

Final 2nd Term Exam
Date: 8 / 6 / 2019
Course Title: Mechanics
Course Code: ENP 012
Duration: 4 hours

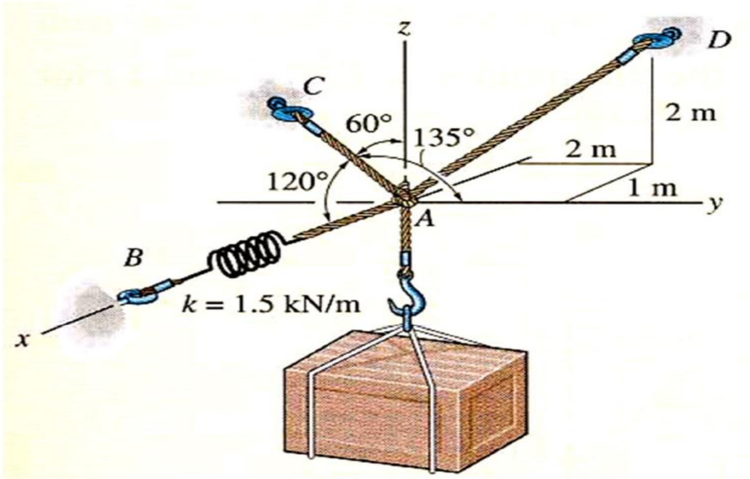
- Answer all the following questions
- Illustrate your answers with sketches when necessary

- Total exam marks : **120**
- Number of questions: **7**
- Number of pages: **4**

Question No. **1** [20 Marks]

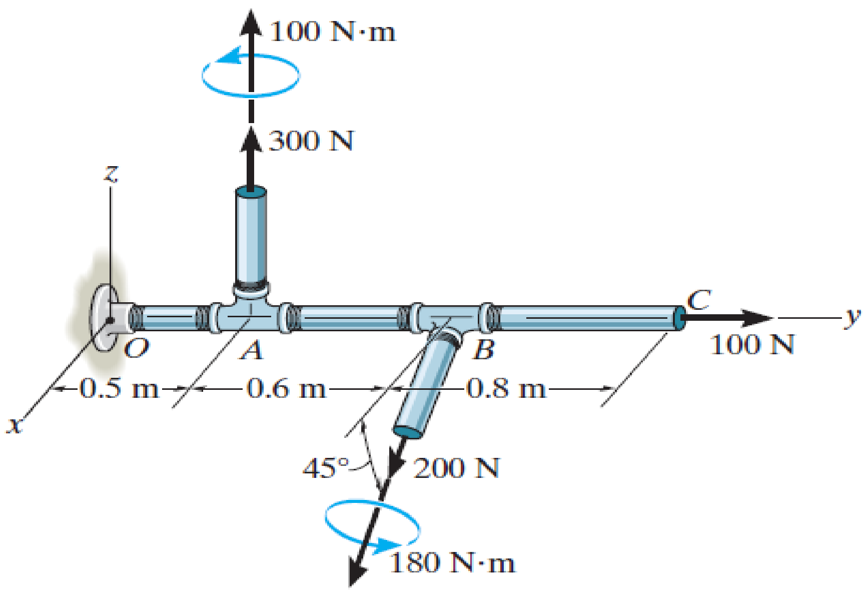
(a) A 100 Kg crate, as shown in the opposite figure, is supported by three cords. One cord has a spring in it. Find the tension in cords AC and AD and the stretch of the spring.

[10 Marks]



(b) Replace the system shown in the opposite figure by a wrench (force and couple at O) and find the intersection of the wrench with x-y plane.

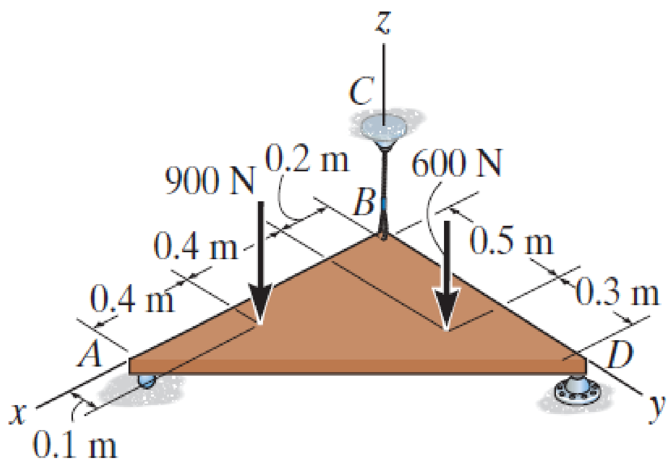
[10 Marks]



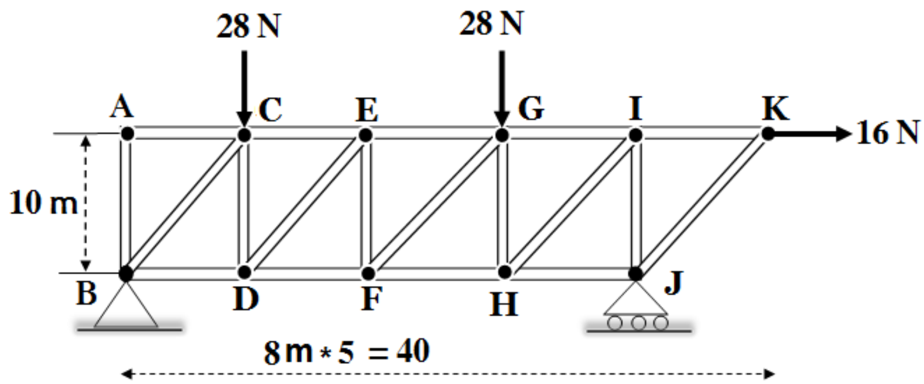
Question No. **2** [25 Marks]

(a) The plate shown in the opposite figure is subjected to a given forces. If it is supported in the horizontal plane by a roller at A, a ball-and-socket joint at D, and a cord at B, determine the reactions at these supports.

[12 Marks]

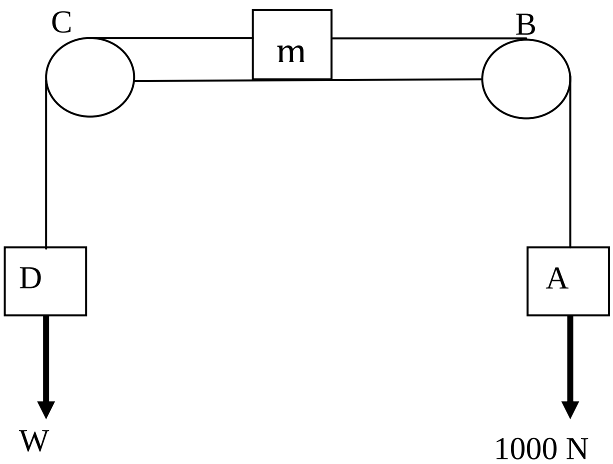


(b) Determine the force in members **EF** and **GI** of the truss shown. State whether the members **EF** and **GI** are in tension or compression. [13 Marks]



Question No. **3** [15 Marks]

A mass (m = 50 kg) is rested on a rough horizontal plane, its coefficient of friction = 0.5. This mass is connected to two horizontal ropes as shown. The first passes over a rough pulley (B), a weight (W) is fixed at the other end of the rope. The second rope passes over a rough pulley at (C). A weight 1000 N is fixed at the other end of the rope. If the coefficient of friction of each pulley = $1/\pi$, find the two limiting values of the weight (W) to keep the mass (m) in equilibrium position . (g = 10 m \ sec^2)



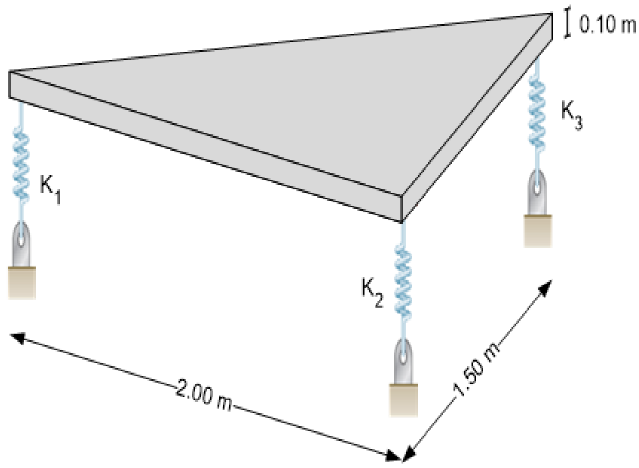
Question No. **4** [18 Marks]

(a) A car moves along a circular track of radius 250 ft such that its speed for a short period of time, $0 \le t \le 4$ is $v = 3(t + t^2)$ ft/sec where t is in seconds. Determine the magnitude of its acceleration when $t = 3$ sec. How far has it traveled in $t = 3$ sec. [6 Marks]

(b) A projectile aimed at a mark which is in a horizontal plane through the point of projection. The projectile falls at (c) meter short of the aim when the angle of projection is α and goes (d) meter too far when the angle of projection is β, show that if the velocity of projection be the same in all cases the correct angle of projection should be:

$$\theta = \frac{1}{2}\sin^{-1}\left\{\frac{c\sin(2\beta) + d\sin(2\alpha)}{c+d}\right\}$$ [12 Marks]

Question No. **5** [12 Marks]



A block with density 1333,333 Kg/m³ is supported by the springs arrangement shown. If $K_1=K_2=K_3=K$ and the motion is simple harmonic motion. If the block moved vertically downward till reach to the equilibrium position given a downward velocity of 30 cm/sec from its equilibrium position and released. Knowing that (g = 10 m/sec²). Please answer the following:

(a) Draw the equivalent system of the shown mechanism (Consider the block as particle).

(b) Find the spring constant such that the periodic time equal to $\pi / 5$ sec.

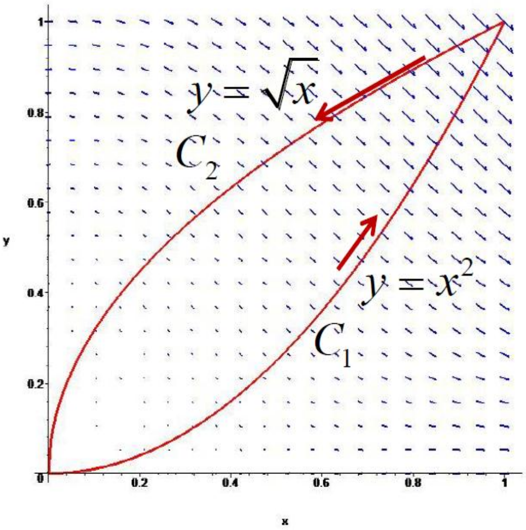
(c) Vertical displacement in each spring at equilibrium position.

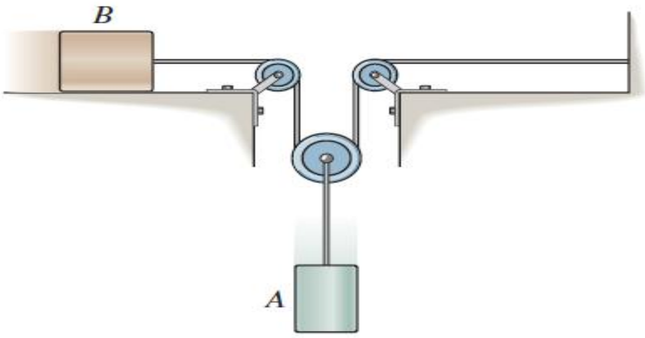
(d) The maximum velocity and maximum acceleration of the block.

Question No. **6** [15 Marks]

(a) Find the work done of $\vec{F}=(x^2+y^2)\vec{i}-(2xy)\vec{j}$ about a closed curve as shown in the opposite figure.

[5 Marks]





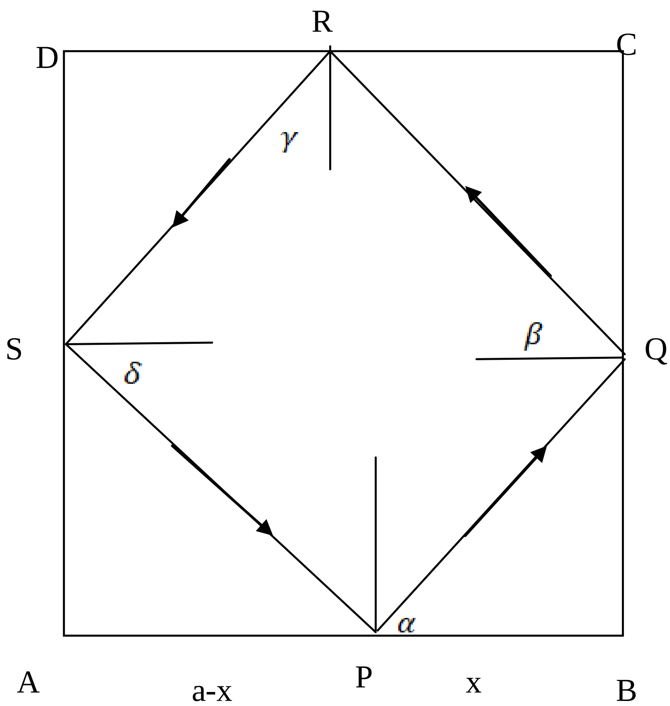
(b) At a given instant the 10-lb block ***A*** is moving downward with a speed of 6 ft/sec. Block ***B*** has a weight of 4 lb, and the coefficient of kinetic friction μ_k = 0.2 between it and the horizontal plane. Determine the speed of block ***A*** after it moved down 2 sec later. Neglect the mass of the cord and pulleys (solve the problem using the work and energy principles).

[10 Marks]

Question No. **7** [15 Marks]

A square table ABCD whose side is (a), has raised edges. A particle of elasticity (e) is projected from a point (P) on AB and hits the sides BC, CD and DA at Q, R and S. Prove that PQ and RS are parallel. If (α) be the angle QPB and PB=x, prove that if the particle returns to P, then:

$$x(1-e)=\alpha(1-e\cot\alpha)\ .$$



Good Luck. Mechanics staff members

Benha University
Faculty of Engineering (Shoubra)
Engineering Mathematics and Physics Department
Preparatory Year

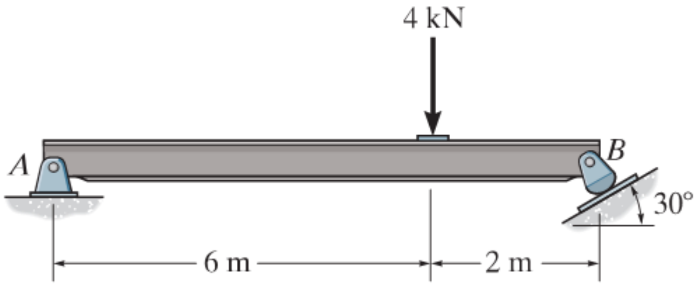
Final 2nd Term Exam
Date: 4 / 6 / 2017
Course Title: Mechanics
Course Code: ENP 012
Duration: 4 hours

- Answer all the following questions
- Illustrate your answers with sketches when necessary
- The exam consists of **2** parts
- Total exam marks : **120**
- No. of questions of part **1**: 3
- Part **1** consists of **2** pages

Model Answer Part 1

Question No. 1 [20 Marks]

(a) In the figure shown determine the reactions at the pin A and the reaction of the rocker B on the beam. Neglect the weight of the beam. [8 Marks]



Solution

$$\sum F_x = 0 \Rightarrow A_x - N_B \sin 30 = 0 \Rightarrow A_x = N_B / 2$$

$$\sum F_y = 0 \Rightarrow A_y + N_B \cos 30 = 4$$

$$\therefore \quad A_y = 4 - \sqrt{3} N_B / 2$$

$$\sum M_B = 0 \Rightarrow 8A_y = 8 \Rightarrow \quad A_y = 1 \quad kN$$

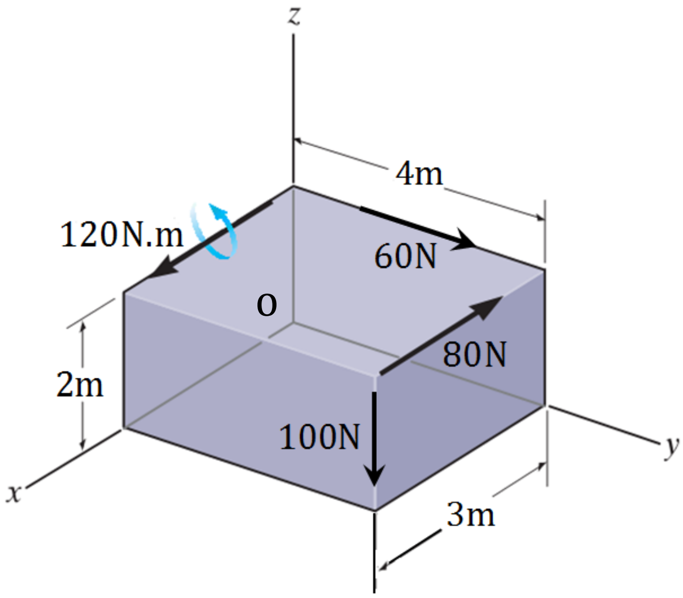
$$\Rightarrow N_B = 2\sqrt{3} = 3.46 \quad kN$$

$$\text{and} \quad A_x = N_B / 2 = 1.73 \quad kN$$

(b) In the shown figure, replace the force and couple moment system acting on the rectangular block by:

(I) A force and a couple at O.

(II) A wrench (specify its pitch and its axis). [12 Marks]



Solution:

$$\overrightarrow{F_R} = -80\hat{i} + 60\hat{j} - 100\hat{k}$$

$$\left|\vec{F}_R\right| = \sqrt{80^2 + 60^2 + 100^2} = 100\sqrt{2} \quad N$$

$$\vec{M}_{R_0} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & 0 \\ 0 & 0 & -100 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 4 & 2 \\ 0 & 60 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 4 & 2 \\ -80 & 0 & 0 \end{vmatrix} + 120\vec{i}$$

$$= -400\vec{i} + 140\vec{j} + 320\vec{k}$$

$$\left|\vec{M}_{\square}\right| = \vec{M}_{R_0} \cdot \vec{F}_R / \left|\vec{F}_R\right| = 42\sqrt{2} \quad N.m$$

$$\vec{M}_{\square} = \left|\vec{M}_{\square}\right| \vec{F}_R / \left|\vec{F}_R\right| = 0.84(-40\hat{i} + 30\hat{j} - 50\hat{k})$$

$$\therefore p = 0.84 \quad m$$

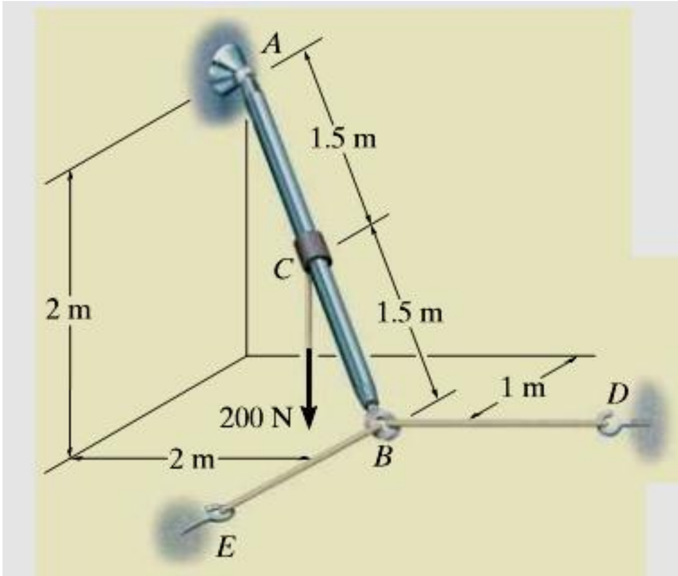
$$\vec{r} = \vec{F}_R \times \vec{M}_{R_0} / \left|\vec{F}_R\right|^2 + \lambda\vec{F}_R = \frac{1}{(100\sqrt{2})^2} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -80 & 60 & -100 \\ -400 & 140 & 320 \end{vmatrix}$$

$$+\lambda(-40\hat{i} + 30\hat{j} - 50\hat{k})$$

$$= (\frac{83}{50} - 80\lambda)\vec{i} + (\frac{164}{50} + 60\lambda)\vec{j} + (\frac{32}{50} - 100\lambda)\vec{k}$$

Question No. 2 [25 Marks]

(a) Rod AB shown in the opposite figure is subjected to the 200 N force. Determine the reactions at the ball-and-socket joint A and the tension in the cables BD and BE. The collar at C is fixed to the rod. [15 Marks]



Solution:

$$\overrightarrow{F_A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\overrightarrow{T_E} = T_E\hat{i}$$

$$\overrightarrow{T_D} = T_D\hat{j}$$

$$\vec{F} = \left(-200\hat{k}\right)N$$

Applying the force equation of equilibrium,

$$\sum\vec{F} = \vec{0};\ \overrightarrow{F_A} = \overrightarrow{T_E} + \overrightarrow{T_D} + \vec{F} = \vec{0}$$

$$\left(A_x + \overrightarrow{T_E}\right)\hat{i} + \left(A_y + T_D\right)\hat{j} + \left(A_z - 200\right)\hat{k} = \vec{0}$$

$$\sum F_x = 0;\ A_x + T_E = 0\ (1)$$

$$\sum F_y = 0;\ A_y + T_D = 0\ (2)$$

$$\sum F_z = 0;\ A_z - 200 = 0\ (3)$$

Summing moments about point A yields

$$\sum\overrightarrow{M_A} = \vec{0};\ \overrightarrow{r_C}\times\vec{F} + \overrightarrow{r_B}\times\left(\overrightarrow{T_E} + \overrightarrow{T_D}\right) = \vec{0}$$

Since $\overrightarrow{r_C} = (1/2)\overrightarrow{r_B}$, then

$$\left(\hat{i}/2 + \hat{j} - \hat{k}\right)\times\left(-200\hat{k}\right) + \left(\hat{i} + 2\hat{j} - 2\hat{k}\right)\times\left(T_E\hat{i} - T_D\hat{j}\right) = \vec{0}$$

Expanding and rearranging terms gives

$$\left(2T_D - 200\right)\hat{i} + \left(-2T_E + 100\right)\hat{j} + \left(T_D - 2T_E\right)\hat{k} = \vec{0}$$

$$\sum M_x = 0; T_D - 100 = 0 \quad (4)$$

$$\sum M_y = 0; -T_E + 50 = 0 \quad (5)$$

$$\sum M_Z = 0; T_D - 2T_E = 0 \quad (6)$$

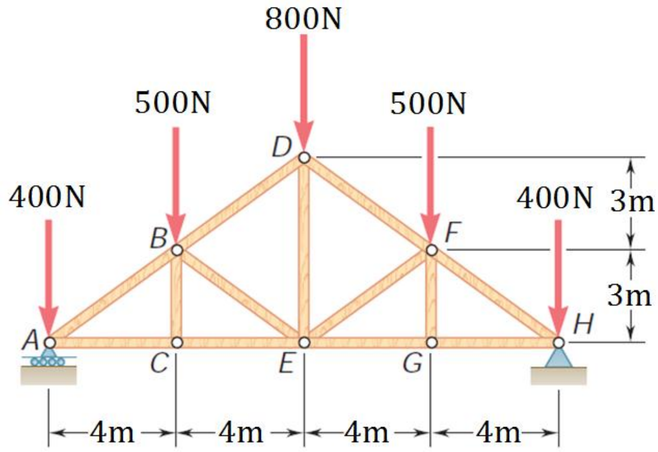
Solving eqs.(1) through (6), we get

$A_x = -50N$, $A_y = -100N$ Ans.

$A_z = 200N$, $T_E = 50N$, $T_D = 100N$ Ans.

The negative sign indicates that A_x and A_y have a sense which is opposite to that shown on the free-body diagram.

(b) Using the method of joints, determine the force in members AB, BD, BE, and CE of the Howe roof truss shown in the opposite figure. State whether each member is in tension or compression. [10 Marks]



$$\sum F_H = 0 \rightarrow H_x = 0, \quad \sum F_V = 0 \rightarrow A_y + H_y = 2600$$
$$\sum M_H = 0 \rightarrow H_y = 1300N, A_y = 1300N$$

At joint A:

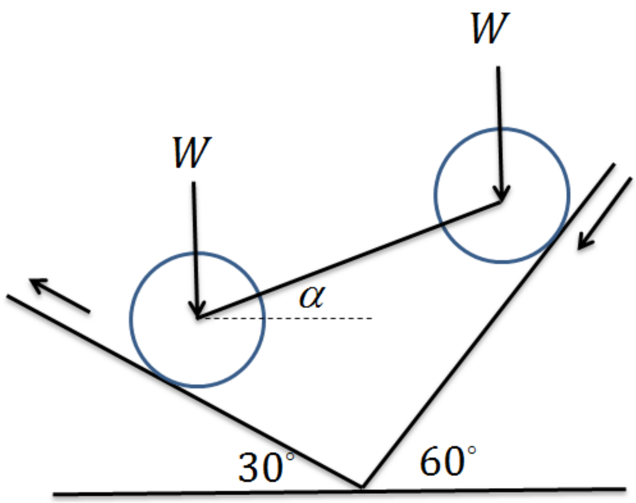
$$\sum F_H = 0 \rightarrow F_{AC} - F_{AB}\cos\theta = 0$$
$$\sum F_V = 0 \rightarrow 1300 - 400 - F_{AB}\sin\theta = 0$$
$$\therefore F_{AB} = 900/\sin\theta = 1500 \quad \text{comp}$$
$$F_{AC} = F_{AB}\cos\theta = 1200 \quad T$$
$$F_{BC} \quad \text{is a zero member}$$
$$\therefore F_{AC} = F_{CE} = 1200 \quad T$$

At joint B:

$$\sum F_H = 0 \rightarrow F_{BE} + F_{BD} = 1500$$
$$\sum F_V = 0 \rightarrow F_{BE} - F_{BD} = -2000/3$$
$$\therefore F_{BE} = 1250/3 \quad \text{comp,}$$
$$F_{BD} = 3250/3 \quad \text{comp.}$$

Question No. 3 [15 Marks]

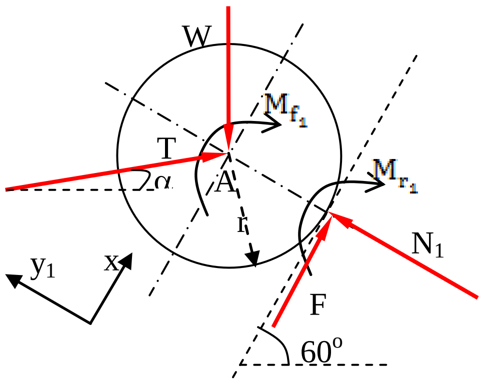
Two wheels each of weight w and radius R, are placed on two rough inclined planes as shown in the opposite figure. The centers of the wheels are joined by a light beam. If the two wheels are impeding to roll find the angle α and the pressure in them if the coefficient of rolling friction = the radius of the circle of friction = R/50.



Solution:

Studying the equilibrium of the right wheel:

Resolve forces in the direction of the plane and the normal to the plane, and take the moment about the center of the right wheel.



$$\sum F_{x_1}=0,\ \sum F_{y_1}=0,\ \sum M_A=0$$

$$F_1-W\cos 30^\circ+T\sin(30^\circ+\alpha)=0 \quad (1)$$

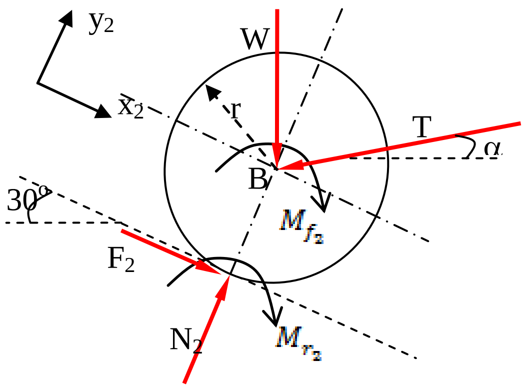
$$N_1-W\sin 30^\circ-T\cos(30^\circ+\alpha)=0 \quad (2)$$

$$F_1 r-M_{f_1}r-T(r/50)-N_1(r/50)=0 \quad (3)$$

Similarly;

Studying the equilibrium of the left wheel :

Resolve and take the moment about the center, we get



$$\sum F_{x_2}=0,\ \sum F_{y_2}=0,\ \sum M_B=0$$

$$F_2+W\sin 30^\circ-T\cos(30^\circ+\alpha)=0 \quad (4)$$

$$N_2+W\cos 30^\circ-T\sin(30^\circ+\alpha)=0 \quad (5)$$

$$F_2 r-T(r/50)-N_2(r/50)=0 \quad (6)$$

Eliminate N1, F1 from eqs. (1), (2), (3).

$$42.5W = T\left[1+\cos\left(30^\circ+\alpha\right)+50\sin\left(30^\circ+\alpha\right)\right] \quad (7)$$

Eliminate N2 and F2 from eqs. (4), (5), (6)

$$25.865W = T\left[50\cos\left(30^\circ+\alpha\right)-\sin\left(30^\circ+\alpha\right)-1\right] \quad (8)$$

Divided (7) & (8), we get:

$$\frac{1+\cos\left(30^\circ+\alpha\right)+50\sin\left(30^\circ+\alpha\right)}{50\cos\left(30^\circ+\alpha\right)-\sin\left(30^\circ+\alpha\right)-1}=1.65$$

$$81.5\cos\left(30^\circ+\alpha\right)-51.65\sin\left(30^\circ+c\right)+2.65=0$$

Which is an equation in $\left(\alpha+30^\circ\right),$ it is solution is given in an approximate method. Neglect the last term, which is relatively small, we get:

$$\tan\left(30^\circ+\alpha\right)=81.50/51.65=1.59$$

$$\therefore 30^\circ+\alpha=58^\circ, \therefore \alpha=28^\circ$$

Substitute into (8) we get: $T = 1.05W$

Benha University
Faculty Of Engineering (Shoubra)
Engineering Mathematics and Physics Department
Preparatory Year

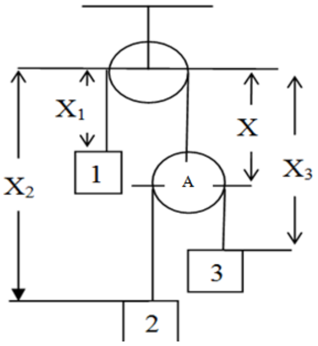
Final 2nd Term Exam
Date: 4 / 6 / 2017
Course Title: Mechanics
Course Code: EMP 012
Duration: 4 hours

- Answer all the following questions
- Illustrate your answers with sketches when necessary
- The exam consists of **2** parts
- Total exam marks : **120**
- No. of questions of part **2**: 3
- Part **2** consists of **2** pages

Model Answer Part 2

Question No. 4 [20 Marks]

(A) In the system shown in the fig. Determine the velocity and acceleration of block 2 at the instant knowing that.



$\mathbf{X^{0}_{1}\uparrow = 6 m/s}$, $\mathbf{X^{00}_{1}\downarrow = 2 m/s^2}$

$\mathbf{X^{0}_{3}\uparrow = 3 m/s}$, $\mathbf{X^{00}_{3}\downarrow = 4 m/s^2}$

Solution

The length of the cord between weights (1) and point "A" is constant and equals

$$l_1 = (x_1 + x + \pi r^2) \qquad (1)$$

The length of the cord between weights (2) and (3) is constant and equals

$$l_2 = (x_2 - 2x + x_3 + \pi r^2) \qquad (2)$$

Differentiate both sides of equations (1 and 2) with respect to time one can get:

$$\dot{x_1} + \dot{x} = 0 \qquad (3)$$

$$\ddot{x_1} + \ddot{x} = 0 \qquad (4)$$

$$\dot{x_2} - 2\dot{x} + \dot{x_3} = 0 \qquad (5)$$

$$\ddot{x_2} - 2\ddot{x} + \ddot{x_3} = 0 \qquad (6)$$

Calling the upward direction positive and substituting $\mathbf{X^{0}_{1}} = 6$ **m/s** into equation (3) we find

$$\dot{x} = -6 \ \mathbf{m/s} \qquad (7)$$

Calling the upward direction positive and substituting $\mathbf{X^{00}_{1} = -2 m/s^2}$ into equation (4) we find

$$\ddot{x} = 2 \ \mathbf{m/s^2} \qquad (8)$$

Calling the upward direction positive and substituting $\mathbf{X^{0}_{3}} = 3$ **m/s** and equation (5) into equation (5) we find

$$\dot{x_2} = -15 \ \mathbf{m/s} \qquad (9)$$

Calling the upward direction positive and substituting $\mathbf{X^{00}_{3}} = -4$ **m/s** and equation (6) into equation (5) we find

$$\ddot{x_2} = +8 \ \mathbf{m/s^2} \qquad (10)$$

(B) A stone is projected with velocity (v) from a height (h) to hit a point in the level at a horizontal distance (R) form the point of projection Show that the angle of projection is given by **$R^2\tan^2\alpha - 2v^2/g\ R\tan\alpha + R^2 - 2hv^2/g = 0$**

Hence deduce that the maximum range on the level for this velocity is

$$R_{\max} = \sqrt{\frac{v^4}{g^2} + \frac{2hv^2}{g}}$$

Solution

The equation of the path for the projectile can be obtained simply as follows:

$$y = x\tan(\propto) - \frac{gx^2}{2v^2\cos^2(\propto)}$$

The point (R, -h) is on the path for the projectile so the following equation can be obtained

$$-h = R\tan(\propto) - \frac{gR^2}{2v^2\cos^2(\propto)}$$

Or

$$-h = R\tan(\propto) - \frac{gR^2}{2v^2}\left(1 + \tan^2(\propto)\right)$$

So the following equation can be obtained:

$$R^2\tan^2(\propto) - \frac{2v^2}{g}R\tan(\propto) + R^2 - \frac{2v^2}{g}h = 0 \quad (1)$$

To get the maximum range $\frac{dR}{d\propto} = 0$ so

Differentiate both sides of equation (1) with respect to α one can get:

$$2R^2\tan(\propto)\sec^2(\propto) - \frac{2v^2}{g}R\sec^2(\propto) = 0$$

$$\tan(\propto) = \frac{2v^2}{g} \quad (2)$$

Substituting equation (2) into equation (1) one can get :

$$R_{\max} = \sqrt{\frac{v^4}{g^2} + \frac{2hv^2}{g}}$$

Question No. 5 [20 Marks]

(A) Find the tangential and normal components of the acceleration of a particle moving according the equations

X= 2t+2 **y= t²-1**

Solution

$$\dot{x} = 2 \qquad \dot{y} = 2t$$

$$\ddot{x} = 0 \qquad \ddot{y} = 2$$

So the magnitude of the velocity is equal:

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = 2\sqrt{1 + t^2}$$

The tangential component of the acceleration is equal to

$$a_t = \dot{v} = \frac{2t}{\sqrt{1+t^2}}$$

So the magnitude of the acceleration is equal to

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = 2$$

Or

$$a = \sqrt{{a_t}^2 + {a_n}^2} = 2$$

So the normal component of the acceleration is equal to

$${a_n}^2 = 4 - \left(\frac{2t}{\sqrt{1+t^2}}\right)^2 = \frac{4+4t^2-2t^2}{1+t^2} = \frac{4}{1+t^2}$$

$$a_n = \frac{2}{\sqrt{1 + t^2}}$$

(B)A 1.75 kg particle moves as function of time as follows: **x = 4 cos (4/3t+π/5)** where distance is measured in meters and time in seconds. Find

(a) The equation of the velocity of this particle as function of time.

(b) The equation of the acceleration of this particle as function of time.

(c) Show that the motion is simple harmonic motion.

(d) The periodic time, frequency, and amplitude of this motion.

(e) The spring constant.

Solution

$$x = 4\cos\left(\tfrac{4}{3}t + \tfrac{\pi}{5}\right) \qquad (1)$$

Differentiate both sides with respect to time one can get

$$v = \tfrac{dx}{dt} = -4 * \tfrac{4}{3}\sin\left(\tfrac{4}{3}t + \tfrac{\pi}{5}\right) \qquad (2)$$

b)

$$v = -4 * \tfrac{4}{3}\sin\left(\tfrac{4}{3}t + \tfrac{\pi}{5}\right) \ (2)$$

Differentiate both sides with respect to time one can get

$$a = \tfrac{dv}{dt} = -4 * \left(\tfrac{4}{3}\right)^2\cos\left(\tfrac{4}{3}t + \tfrac{\pi}{5}\right) \qquad (3)$$

c)

From equation (1) and (3) one can get

$$a = -\left(\tfrac{4}{3}\right)^2 x \qquad (4) \qquad \text{The motion is a S.H.M}$$

d)

$\because a = -\omega^2 x$ Comparing with equation (4)

$$\omega^2 = \left(\tfrac{4}{3}\right)^2 \qquad \text{So,} \qquad \omega = \tfrac{4}{3}\ rad/sec$$

$$\tau = \frac{2\Pi}{\omega} = \frac{6\Pi}{4} sec$$

$$f = \frac{1}{\tau} = \frac{4}{6\Pi}\ s^{-1}$$

The amplitude can be obtained by substituting by v=0 in (2) get x=A

$$0 = -4 * \tfrac{4}{3}\sin\left(\tfrac{4}{3}t + \tfrac{\pi}{5}\right)$$

$$\text{So,}\ t = 9\tfrac{\pi}{40}\ sec$$

$$\text{Substituting}\ t = 9\tfrac{\pi}{40}\ \text{in (1)}$$

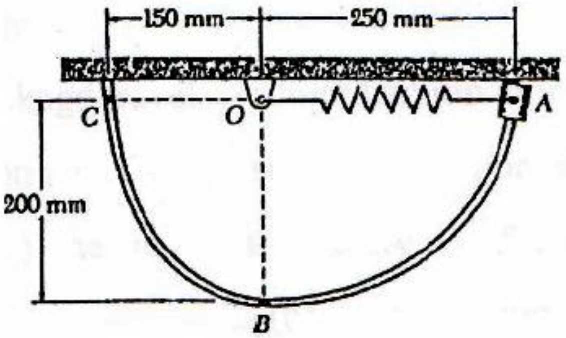
$$x = A = 4\cos\left(\tfrac{4}{3} * 9\tfrac{\pi}{40} + \tfrac{\pi}{5}\right) = 4\ m$$

e)

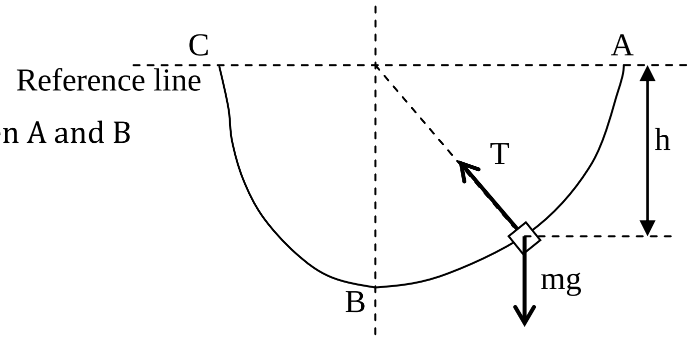
$$\because \omega = \sqrt{\frac{k}{m}} \qquad \therefore k = m\,\omega^2 = 1.75 * \left(\tfrac{4}{3}\right)^2 = 3.11\ N/m$$

Question No. 6 [20 Marks]

(A) A slider of mass **2 kg** attached to a spring of stiffness **600 N/m**, the spring is un-deformed when the collar is at C. If the collar is released from rest at A, determine the velocity of the slider as it passes through B and C.



Solution



Apply the law of conservation of energy between A and B

$$U_A + K_A = U_B + K_B$$

$$\left(mgh_A + \frac{1}{2}Ks_A^2\right) + \frac{1}{2}mv_A^2 = \left(mgh_B + \frac{1}{2}Ks_B^2\right) + \frac{1}{2}mv_B^2$$

$$\left(0 + \frac{1}{2} * 600 * (0.25 - 0.15)^2\right) + 0 = \left(-2 * 9.81 * 0.2 + \frac{1}{2} * 600 * (0.2 - 0.15)^2\right) + \frac{1}{2} * 2 * v_B^2$$

$v_B = 2.485$ m/sec

Apply the law of conservation of energy between A and C

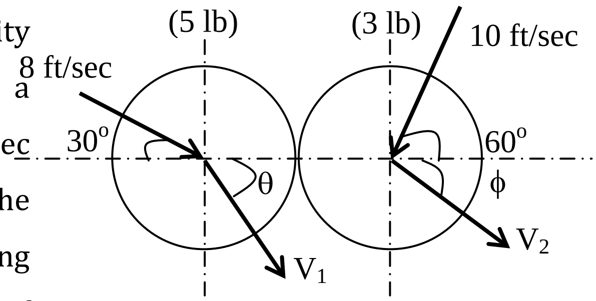
$$U_A + K_A = U_C + K_C$$

$$\left(mgh_A + \frac{1}{2}Ks_A^2\right) + \frac{1}{2}mv_A^2 = \left(mgh_C + \frac{1}{2}Ks_C^2\right) + \frac{1}{2}mv_C^2$$

$$\left(0 + \frac{1}{2} * 600 * (0.25 - 0.15)^2\right) + 0 = (0 + 0) + \frac{1}{2} * 2 * v_C^2$$

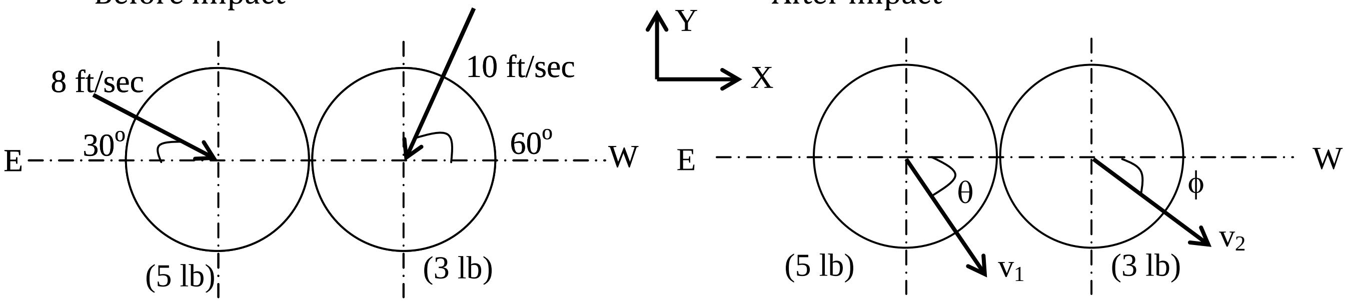
$v_C = 1.732$ m/sec

(B) A smooth sphere of mass **(5 lb)** moves with velocity 8 ft/sec in a direction **30° N of W**, collies with a sphere of mass **(3 lb)** moves with velocity 10 ft/sec in a direction **30° E of N** as shown in figure. The impact takes place in such a way that the line joining the centers of the spheres is E and W at the time of impact. Determine the velocities of the two spheres after impact. **(e=0.5)**



Solution

Before impact After impact



Apply the law of conservation of momentum

$m_1 u_1 cos\alpha + m_2 u_2 cos\beta = m_1 v_1 cos\theta + m_2 v_2 cos\phi$

$5 * 8 * cos30 - 3 * 10 * cos60 = 5 * v_1 cos\theta + 3 * v_2 cos\phi$

$19.641 = 5 * v_1 cos\theta + 3 * v_2 cos\phi$ (a)

Apply the Newton's experimental law

$$\frac{v_1 cos\theta - v_2 cos\phi}{u_1 cos\alpha - u_2 cos\beta} = -e$$

$$\frac{v_1 cos\theta - v_2 cos\phi}{8 * cos30 + 10 * cos60} = -0.5$$

$v_1 cos\theta - v_2 cos\phi = -5.964$ (b)

Multiply equation (b) by 3 then add to equation (a)

$v_1 cos\theta = 0.218625$ (c)

Since, the two spheres are smooth, i.e. there is no friction, then the impact only occur in E W axis (x-axis) only.

$v_1 sin\theta = 8 * sin30$ (d)

$v_2 sin\phi = 10 * sin60$ (e)

From (c) and (d)

$tan\theta = 18.296 \qquad \theta = 86.872^0$

$v_1 = 4.01$ ft/sec

From (c) and (b)

$v_2 cos\phi = 6.182625 \qquad \text{(f)}$

From (f) and (e)

$tan\phi = 1.4007 \qquad \phi = 54.477^0$

$v_1 = 10{,}641$ ft/sec